Reports of the Department of Geodetic Science
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A FORTRAN IV PROGRAM FOR THE DETERMINATION OF THE ANOMALOUS POTENTIAL USING STEPWISE LEAST SQUARES COLLOCATION

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Abstract

The theory of sequential least squares collocation, as applied to the determination of an approximation $\tilde{T}$ to the anomalous potential of the Earth T, and to the prediction and filtering of quantities related in a linear manner to T, is developed.

The practical implementation of the theory in the form of a FORTRAN IV program is presented, and detailed instructions for the use of this program are given.

The program requires the specification of (1) a covariance function of the gravity anomalies and (2) a set of observed quantities (with known standard deviations).

The covariance function is required to be isotropic. It is specified by a set of empirical anomaly degree-variances all of degree less than or equal to an integer I and by selecting the anomaly degree-variances of degree greater than I according to one of three possible degree-variance models. The observations may be potential coefficients, mean or point gravity anomalies, height anomalies or deflections of the vertical. A filtering of the observations will take place simultaneously with the determination of $\tilde{T}$.

The program may be used for the prediction of height anomalies, gravity anomalies and deflections of the vertical. Estimates of the standard error of the predicted quantities may be obtained as well.

The observations may be given in a local geodetic reference system. In this case parameters for a datum shift to a geocentric reference system must be specified. The predictions will be given in both the local and the geocentric reference system.

$\tilde{T}$ may be computed stepwise, i.e. the observations may be divided in up to three groups. (The limit of three is only attained when potential coefficients are observed, in which case these quantities will form the first set of observations.) Each set of observations will determine a harmonic function and $\tilde{T}$ will be equal to the sum of these functions.

The function $\tilde{T}$ determined by the program will be a (global or local) solution to the problem of Bjerhammar, i.e., it will be harmonic outside a sphere enclosed in the Earth, and it will agree with the filtered observations.

Foreword

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1. Introduction.

The theory of least squares collocation has been discussed extensively by Krarup (1969), Moritz (1972, 1973), Lauritzen (1973), Grafarend (1973) and Tscherning (1973). Collocation was originally introduced by Krarup (1969) as a method for the determination of the anomalous potential using different kinds of observations. Primarily Moritz (1972) has extended the theory to a wider field. This report will only consider the use of least squares collocation for the determination of the anomalous potential, T and the estimation of quantities dependent on T.

We will regard the problem of the determination of T as being equivalent to the solution of the Bjerhammar-problem, i. e. the determination of a function, harmonic outside a sphere totally enclosed in the Earth and regular at infinity, which agrees with observed values of e. g. gravity anomalies and deflections of the vertical.

It is not required, that the observations and the solution agree exactly. The observations will contain a certain amount of "noise", the magnitude of which is specified by an estimated standard error.

Least squares collocation will filter out some of this noise, and the solution will agree exactly with the filtered observations.

Having determined a solution, $\tilde{T}$, to the Bjerhammar problem, this function can naturally he used to compute geoid heights (or more correctly, height anomalies), gravity anomalies or deflections of the vertical in points in the set of harmonicity. Hence, by solving the Bjerhammar problem, we have implicitly also solved e. g. the problems of interpolation or extrapolation (prediction) of gravity anomalies or deflections and the problem of astrogeodetic or astrogravimetric geoid computation.

An Algol-program, which used this approach was published in Tscherning (1972). The program could only handle a very limited amount of observations. In the FORTRAN IV program presented in this report, we have taken advantage of the availability of a computer (IBM System 370), which has large core storage and fast peripherial units (disks), so that very large amounts of data can be treated. Thus, the use of FORTRAN IV, which does not have variable dimensioning of arrays, has required that certain (arbitrary) limits have been put on e.g. the number of observations, which the program can handle.

In section 2 we will present the basic equations of least squares collocation as applied to the Bjerhammar problem. We will also discuss the method of stepwise collocation, which differs somewhat from the p ocedure described by Moritz (1973). All the observed quantities must be in the same reference system. This requirement is discussed in section **3**. The main lines of function of the FORTRAN IV program is described in section 4. The most important details are given in the following section, which especially discusses the subprograms used. Input and output options are described in sections G and 7 respectively, and the final section 8 contains some recommendations and conclusions. The FORTRAN program, an input and an output example are contained in an appendix.

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References are given by author name and year, with one exception: Heiskanen and Moritz, Physical Geodesy, will be referenced only by PG, because references to this book occur frequently.

2. The Basic Equations

There are two ways of approach to least squares collocation. A mathematical (functional analytic) and a statistical. The mathematical approach is the most well founded and without dark spots. But its appreciation requires a mathmatical background, which not yet is common among geodesists. The statistical approach is with a first glance less difficult and gives a sufficient insight. This means, that a geodesist, well educated in the theory and application of least squares adjustment, will be able to use the method.

We will, without hereby having questioned the intellectual ability of the reader, use the statistical approach in the following presentation of the basic equations of least squares collocation.

2.1 Least squares collocation.

Let us suppose, that T is an element of a sample space H of functions harmonic outside a sphere totally enclosed in the Earth. We will denote the probability measure of H by Φ. Let the random variables Y_P be the mappings, which relate a function in H to the value of the function in the point P, i.e. $Y_P(T) = T(P)$. The variables Y_P will then form a stochastic process with the set of harmonicity as index set (provided P fulfills some basic requirements see e.g. Grafarend (1973)). The covariance between two random variables Y_P, Y_Q will be denoted $\mathrm{cov}(T_P, T_Q)$, because $Y_P(T) = T(P)$ and $Y_Q(T) = T(Q)$. It is equal to:

$$\text{(1)} \qquad \mathrm{cov}(T_P, T_Q) = \int_H Y_P(T) \cdot Y_Q(T)\, d\Phi$$

We will require, that the variance $\mathrm{cov}(T_P, T_Q)$ is finite.

Example. Following Meissl (1971), the probability measure Φ may be defined by specifying the distribution of the random variables, $Y_{\ell m}$, which maps T into its coefficient $v_{\ell m}$ in a development of T as a series in solid spherical harmonics. Let us suppose that this random variable has a Gaussian distribution with mean value zero and variance $\sigma_\ell(T,T)/\sqrt{2\ell+1}$, only depending of the degree ℓ, we will then have

$$\mathrm{cov}(T_P, T_Q) = \sum_{\ell=0}^{\infty} \sigma_\ell(T,T) \cdot \left(\frac{R^2}{r \cdot r'}\right)^{\ell+1} P_\ell(\cos\psi),$$

where ψ is the spherical distance between P and Q, r and r' the distance of P and Q from the origin, R the radius of the (Bjerhammar)-sphere bounding the set of harmonicity and $P_\ell(\cos\psi)$ the Legendre polynomial of degree ℓ.

The constants $\sigma_\ell(T,T)$ are called the (potential) degree variances. The covariance function will be isotropic, i. e. invariant with respect to rotations of the pair of points P and Q around the origin*

From the random variables Y_P we may form a second order stochastic field. This field consists of all random variables, which are linear combinations or limits of linear combinations of a finite number of these random variables and which have finite variance, (cf. e.g. Parzen (1967), page 260).

Their covariances can all be derived from the covariance (1). Let us regard

$$Y = a_1 Y_{P_1} + a_2 Y_{P_2}, \qquad Y(T) = a_1 T(P_1) + a_2 T(P_2).$$

Then

$$\mathrm{cov}(Y(T), T_Q) = \int_H Y(T) \cdot Y_Q(T) d\Phi = a_1 \int_H Y_{P_1}(T) \cdot Y_Q(T) d\Phi$$

$$+ a_2 \int_H Y_{P_2}(Y) \cdot Y_Q(T) d\Phi = a_1 \mathrm{cov}(T_{P_1}, T_Q) + a_2 \mathrm{cov}(T_{P_2}, T_Q).$$

And generally for quantities s_i and s_j, where

$$(2) \quad s_i = Y_i(T), \qquad s_j = Y_j(T)$$

we have

$$(3) \quad \mathrm{cov}(s_i, s_j) = Y_i(\mathrm{cov}(T_P, s_j)) = Y_i(Y_j(\mathrm{cov}(T_P, T_Q))).$$

(We have here implicitly presupposed, that cov (T_P, s_j) regarded as a function of P and $Y_j(\mathrm{cov}(T_P, T_Q))$ regarded as a function of Q, are elements of the sample space H, i.e. that they are harmonic. This will be proved below).

The equation (3) is the so called law of propagation of covariances, Moritz (1972, page 97).

We will denote

(4) $c_{ij} = \mathrm{cov}(s_i, s_j)$, $C = \{c_{ij}\}$ a q x q matrix,

(5) $c_{Pi} = \mathrm{cov}(T_P, s_i)$, $C_P = \{c_{Pi}\}$ a q-vector and

(6) $c_{si} = \mathrm{cov}(s, s_i)$, $C_s = \{c_{si}\}$ a q-vector.

(We will below use subscripted quantities in brackets, { } to denote vectors or matrices. In case the limit(s) of the subscript(s) are not obvious, the upper limit(s) will be indicated by subscripts, i. e. $C = \{c_{ij}\}_{q \times q}$).

We will have to regard one more kind of random quantities (independent of the above discussed), namely the random noise, n. A random variable will be associated with each of the random variables Y_s. They are all supposed to be Gaussian distributed with mean value zero, known variance (denoted σ_s^2) and uncorrelated. The covariance matrix, which hence is a diagonal matrix, will be denoted $D = \{d_{ij}\}$, $d_{ii} = \sigma_s^2$

Following Moritz (1972), the basic equation of "observation" is

(7) $x = AX + s' + n$,

where x is the measurement or observation, s' the corresponding "signal" and n is the noise. x, s' and n are q-vectors, where q is the number of observations. The n-vector X comprises n parameters, and A is a known q x m matrix.

Let us now assume, that we want to estimate the outcome s of a stochastic variable Y_s, given a set of observed quantities x. Denoting $\overline{C} = C + D$ we obtain from Moritz (1972, eq. (2-38) and (2-35))

(8) $\tilde{s} = C_s^T \overline{C}^{-1}(x - AX)$

(9) $\tilde{X} = (A^T \overline{C}^{-1} A)^{-1} A^T \overline{C}^{-1} x$

where the superscript T means transposition.

The corresponding estimate of the error of estimation m_s^2 (of s) and E_{XX} (of X) are, cf. Moritz (1972, eq. (3-38) and (3-33))

(10) $m_s^2 = C_{ss} - C_s^T \bar{C}^{-1} C_s + h_s^T A E_{XX} A^T h_s$,

(11) $E_{XX} = (A^T \bar{C}^{-1} A)^{-1}$

with $h_s = C_s^T \bar{C}^{-1}$ and C_{ss} is the variance of s.

The program presented in this report can only handle the non-parametric (i.e. X= 0) case. But the general equations are presented here, so that we later on can point out the main changes, which will have to be made in order to incorporate the parameters X.

The special case we will consider here can then be described by the following equations:

(12) $\tilde{s} = C_s^T \bar{C}^{-1} x$ and

(13) $m_s^2 = C_{ss} - C_s^T \bar{C}^{-1} C_s$

The filtered observations $\tilde{s}'$ are obtained from (12) by substituting C^T for C_s^T.

The equations (12) and (13) differ from the equations given by PG (eq. (7-63) and (7-64)) only in that $\bar{C}$ has been substituted for C and that we are not restricted to consider only gravity anomalies.

The quantities we want to consider here are potential coefficients, gravity anomalies, deflections of the vertical and height anomalies. They are all (at least in spherical approximation) expressible as either linear combinations or limits of linear combinations of values of the anomalous potential. We will presuppose, that the variances of the corresponding stochastic variables all are finite. Equation (3) is hence valid for these kinds of quantities.

The value of the Laplace operator A, applied on T and evaluated in a point P in the set of harmonicity,

$$\Delta T_P = 0$$

is related to a stochastic variable, $Y_{\Delta T_P}$. This variable will also belong to the stochastic field (variance zero) and we will have for an arbitrary stochastic variable Y_s:

(14) $\mathrm{cov}(s, \Delta T_P) = \Delta(\mathrm{cov}(s, T_P)) = 0.$

Hence, the covariance between a quantity s and the value of the anomalous potential in P is a harmonic function (regarded as a function of P).

Let us now assume, that we want to estimat the value of T in a point P from a set of observations $x = \{x_i\}$, $i = 1, \ldots, q$. We then have from (12),

$$(15)\quad \tilde{T}(P) = C_P^T \bar{C}^{-1} x = \{\mathrm{cov}(T_P, s_i)\}^T \{\mathrm{cov}(s_i, s_j) + d_{ij}\}^{-1} \{x_j\}.$$

Introducting the solution vector

$$(16)\quad b = \{b_i\} = \bar{C}^{-1} x$$

we have

$$(17)\quad \tilde{T}(P) = C_P^T b = \{\mathrm{cov}(T_P, s_i)\}^T \{b_i\} = \sum_{i=1}^{q} \mathrm{cov}(T_P, s_i) b_i$$

Using (11) we see that

$$(18)\quad \Delta_P \tilde{T} = \sum_{i=1}^{q} \Delta_P (\mathrm{cov}(T_P, s_i)) \cdot b_i = 0$$

i. e. $\tilde{T}(P)$ is a harmonic function.

By also requiring, that the functions in the sample space H are regular at infinity, it can be shown, that $\tilde{T}(P)$ is regular at infinity as well.

We have then obtained a solution to the problem of Bjerhammar, if we can prove, that the $\tilde{s}_i = s_i = x_i$ for $\sigma_{s_i}^2$ (or d_{ii}) = 0. But this is easily seen, because

$$(19)\quad \tilde{s}_i = Y_{s_i}(\tilde{T}(P)) = \sum_{j=1}^{q} Y_{s_i}(\mathrm{cov}(T_P, s_j)) \cdot b_j$$

$$= \{\mathrm{cov}(s_i, s_j)\}^T \{\mathrm{cov}(s_j, s_k)\}^{-1} \{x_k\} = \{0, \ldots, 0, 1, \ldots, 0\}^T \{x_k\} = s_i.$$

(1 at i'th positions)

This fact makes available an easy test of a collocation program. The used observations are predicted and it is checked, that the predictions agree with the observed values (and that the estimates of the error of prediction are zero).

2.2 Equations for the covariances of and between gravity anomalies, deflections of the vertical, height anomalies and potential coefficients.

The relation (3) between the signal and the anomalous potential has been given in Tscherning and Rapp (1974), eq. (30)-(33))in spherical approximation for gravity anomalies, height anomalies and deflections of the vertical. We have for the height anomaly in P

(20) $\zeta = T(P)/\gamma,$

the latitude component of the deflection of the vertical

(21) $\xi = -D_{\varphi}\,T(P)/(\gamma \cdot r),$

the longitude component of the deflection of the vertical

(22) $\eta = -D_{\lambda}T(P)/(\gamma \cdot r \cdot \cos\varphi),$

the point (free-air) gravity anomaly

(23) $\Delta g = -D_r T(P) - \dfrac{2}{r}\,T(P)$

and the mean (free-air) gravity anomaly

(24) $\overline{\Delta g} = \dfrac{1}{A}\int \Delta g\, dA,$

where r is the distance from the origin, φ the latitude, X the longitude, y the reference gravity and **A** the area over which the mean gravity value is computed.

We may, as explained in Tscherning and Rapp (1974, section 10) represent mean gravity anomalies by point anomalies in a certain height above the center of the area **A.** For this reason we will not in the following distinguish between mean and point gravity anomalies. The program is able to use all the quantities (20)-(24)as observed quantities for the computation of $\tilde{T}$. The same kind of quantities may also be predicted by the program.

One more kind of quantities, potential coefficients, can be used, though only as observed quantities. The given coefficients will generally be the coefficients of the potential of the Earth, W, expanded in spherical harmonics and not the coefficients of the anomalous potential. Denoting the normal potential by U we have

$$T(P) = W(P) - U(P)$$

and for W and U expanded in fully normalized spherical harmonics

$$(25)\quad W(P)=\frac{kM}{r}\left(1+\sum_{\ell=1}^{\infty}\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}\overline{P}_{\ell m}(\cos\theta)(\overline{S}_{\ell m}\cdot\sin m\lambda+\overline{C}_{\ell m}\cdot\cos m\lambda)\right)$$

$$+\frac{\omega^2}{2}(r\cdot\sin\theta)^2 \qquad \text{and}$$

$$(26)\quad U(P)=\frac{kM}{r}\left(1-\sum_{\ell=1}^{\infty}\left(\frac{a}{r}\right)^{2\ell}\cdot\frac{J_{2\ell}}{\sqrt{4\ell+1}}\overline{P}_{2\ell}(\cos\theta)\right)+\frac{\omega^2}{2}(r\sin\theta)^2,$$

where ω is the speed of rotation of the Earth, $\theta=90^\circ-\varphi$, kM the product of the gravitational constant and the total mass of the Earth, the coefficients $\overline{S}_{\ell m}$ and $\overline{C}_{\ell m}$ the potential coefficients, and $\overline{P}_{\ell m}(\cos\theta)$ an associated Legendre polynomial, normalized so that

$$(27)\quad \frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\left(\overline{P}_{\ell m}(\cos\theta)\begin{Bmatrix}\cos m\lambda\\ \sin m\lambda\end{Bmatrix}\right)^2\sin\theta\, d\theta\, d\lambda=1.$$

The coefficients $J_{2\ell}$ in (26) are given in PG, (eq. (2-92)).

For the potential coefficients we then have the following equation

$$(28)\quad \left.\begin{matrix}a\cdot kM\cdot\overline{S}_{\ell m}\\ a\cdot kM\cdot\overline{C}_{\ell m}\end{matrix}\right\}=\frac{1}{4\pi}\iint_{\omega}(T+U)\cdot\overline{P}_{\ell m}(\cos\theta)\begin{Bmatrix}\sin m\lambda\\ \cos m\lambda\end{Bmatrix}d\omega,$$

where ω is the surface of the sphere with radius equal to the semi major axis a and with center in the origin. (We are now denoting two quantities by ω, but since they are used in a different context, we hope, that no confusion is caused).

In the program it is possible to use one of three different kinds of (isotropic) covariance functions, which we below will distinguish by a subscript k, k = 1, 2 or 3. They are all specified by a so called anomaly degree-variance model, i.e. by the coefficients $\sigma_{k,\ell}(\Delta g,\Delta g)$ of degree ℓ greater than a constant I of the covariance function of the gravity anomalies developed in a Legendre series:

$$(29)\quad \mathrm{cov}_k(\Delta g_P,\Delta g_Q)=\sum_{\ell=0}^{I}\hat{\sigma}_{\ell}(\Delta g,\Delta g)\left(\frac{R^2}{r\cdot r'}\right)^{\ell+2}P_{\ell}(\cos\psi)+\sum_{\ell=I+1}^{\infty}\sigma_{k,\ell}(\Delta g,\Delta g)$$

(29) (cont'd)
$$\cdot\left(\frac{R^2}{r\cdot r'}\right)^{\ell+2} P_\ell(\cos\psi),$$

where r' is the distance of Q from the origin, R the radius of the Bjerhammar sphere, ψ = the spherical distance between P and Q and $\hat{\sigma}_\ell(\Delta g, \Delta g)$ are empirically determined coefficients.

The three different kinds of covariance functions correspond to three of the five anomaly degree-variance models discussed in Tscherning and Rapp (1974, section 8). The models are for k = 1,2 and 3:

$$\text{(30)} \quad \sigma_{1,\ell}(\Delta g, \Delta g) = \frac{A_1(\ell-1)}{\ell}, \quad \ell > I \geq 1,$$

$$\text{(31)} \quad \sigma_{2,\ell}(\Delta g, \Delta g) = \frac{A_2(\ell-1)}{\ell-2}, \quad \ell > I \geq 2 \quad \text{and}$$

$$\text{(32)} \quad \sigma_{3,\ell}(\Delta g, \Delta g) = \frac{A_3(\ell-1)}{(\ell-2)(\ell+B)}, \quad \ell > I \geq 2$$

where A_k, k = 1, 2 and 3 are constants of dimension mgal^2 and B is a positive integer (denoted IK in the program).

A part of the specification of the degree-variance model is the value of the radius of the Bjerhammar sphere. In the program this quantity is specified through the ratio R/R_e, where R_e is a mean Earth radius, (equal to 6371.0 km in the program).

The covariance function of the anomalous potential may be expanded in a similar way in a Legendre series, cf. Tscherning and Rapp (1973, eq. (144)),

$$\text{(33)} \quad \mathrm{cov}_k(T_P, T_Q) = \sum_{\ell=0}^{I} \hat{\sigma}_\ell(T,T) s^{\ell+1} P_\ell(t) + \sum_{\ell=I+1}^{\infty} \sigma_{k,\ell}(T,T) s^{\ell+1} P_\ell(t)$$

where $t = \cos\psi$, $s = \dfrac{R^2}{r\cdot r'}$ and

$$\text{(34)} \begin{cases} \hat{\sigma}_\ell(T,T) = \dfrac{R^2}{(\ell-1)^2}\,\hat{\sigma}_\ell(\Delta g, \Delta g), & \ell \leq I, \quad \ell > 1 \\[2ex] \sigma_{k,\ell}(T,T) = \dfrac{R^2}{(\ell-1)^2}\,\sigma_{\ell,k}(\Delta g, \Delta g), & \ell > I. \end{cases}$$

(Degree-variances of degree zero and one will always be equal to zero.)

We rearrange (33):

$$(35)\quad \operatorname{cov}_k(T_P, T_Q) = \sum_{\ell=0}^{I} (\hat{\sigma}_\ell(T,T) - \sigma_{k,\ell}(T,T)) s^{\ell+1} P_\ell(t)$$

$$+ \sum_{\ell=0}^{\infty} \sigma_{k,\ell}(T,T) s^{\ell+1} P_\ell(t).$$

Denoting

$$(36)\quad \epsilon_{k,\ell}(T,T) = \hat{\sigma}_\ell(T,T) - \sigma_{k,\ell}(T,T) \quad , \quad \ell \leq I,$$

$$(37)\quad \operatorname{cov}_k^{I}(T_P, T_Q) = \sum_{\ell=0}^{I} \epsilon_{k,\ell}(T,T) s^{\ell+1} P_\ell(t) \quad \text{and}$$

$$(38)\quad \operatorname{cov}_k^{O}(T_P, T_Q) = \sum_{\ell=0}^{\infty} \sigma_{k,\ell}(T,T) s^{\ell+1} P_\ell(t)$$

we will have

$$(39)\quad \operatorname{cov}_k(T_P, T_Q) = \operatorname{cov}_k^{I}(T_P, T_Q) + \operatorname{cov}_k^{O}(T_P, T_Q).$$

From this covariance function all the other covariance functions can be derived using the "law of propagation of covariances", eq. (3) and the equations (20)-(24) and (28) relating the observed quantities to the anomalous potential.

Due to the linear relationship (39) we generally have for two arbitrary random variables Y_i and Y_j

$$(40)\quad \operatorname{cov}_k(s_i, s_j) = \operatorname{cov}_k^{I}(s_i, s_j) + \operatorname{cov}_k^{O}(s_i, s_j)$$

where $s_i = Y_i(T)$ and $s_j = Y_j(T)$.

For either s_i equal to Δg_P or ζ_P and s_j equal to Δg_Q or ζ_Q can the quantity $\operatorname{cov}_k^{O}(s_i, s_j)$ be represented by a closed expression, cf. Tscherning and Rapp (1974, equations (105)-(107), (115)-(117) and (130)-(132)).

For the other part, $\mathrm{cov}_k^!(s_i, s_j)$ we have (from Tscherning and Rapp 11974 equations (145)-(150)and (50)))

$$(41)\quad \mathrm{cov}_k^!(\zeta_P, \zeta_Q) = \sum_{\ell=0}^{!} \epsilon_{k,\ell}(T,T)\frac{1}{\gamma\cdot\gamma'}\cdot s^{\ell+1} P_\ell(t)$$

$$(42)\quad \mathrm{cov}_k^!(\zeta_P, \Delta g_Q) = \sum_{\ell=0}^{!} \epsilon_{k,\ell}(T,\Delta g) s^{\ell+1} P_\ell(t)\cdot\frac{R}{r'\cdot\gamma}$$ and

$$(43)\quad \mathrm{cov}_k^!(\Delta g_P, \Delta g_Q) = \sum_{\ell=0}^{!} \epsilon_{k,\ell}(\Delta g,\Delta g) s^{\ell+2} P_\ell(t)$$

with

$$\epsilon_{k,\ell}(T,\Delta g) = \frac{(\ell-1)}{R}\cdot\epsilon_{k,\ell}(T,T) \quad \text{and}$$

$$\epsilon_{k,\ell}(\Delta g,\Delta g) = \frac{(\ell-1)^2}{R^2}\,\epsilon_{k,\ell}(T,T).$$

Covariance functions, $\mathrm{cov}(s_i, s_j)$, where either s_i or s_j is a deflection component can not explicitly be found in Tscherning and Rapp (1974). But using the equations (20)-(24)we get ($K = K(P,Q) = \mathrm{cov},(T_P, T_Q)$):

$$(44)\quad \mathrm{cov}(\zeta_P, \xi_Q) = -D_\varphi K/(\gamma\cdot\gamma'\cdot r')$$

$$= -D_\varphi t\cdot D_t K/(\gamma\cdot\gamma'\cdot r'),$$

$$(45)\quad \mathrm{cov}(\zeta_P, \eta_Q) = -D_\lambda K/(\gamma\cdot\gamma'\cdot r'\cdot\cos\varphi')$$

$$= -D_\lambda t\cdot D_t K/(\gamma\cdot\gamma\cdot r'\cdot\cos\varphi')$$

$$(46)\quad \mathrm{cov}(\xi_P, \Delta g_Q) = -D_\varphi\cdot(\mathrm{cov}(\Delta g_P, T_Q))/(\gamma\cdot r)$$

$$= -D_\varphi t\cdot D_t(\mathrm{cov}(\Delta g_P, T_Q))/(\gamma\cdot r)$$

(47) $\mathrm{cov}(\eta_P, \Delta g_Q) = -D_\lambda(\mathrm{cov}(\Delta g_P, T_Q))/(\gamma \cdot r \cdot \cos\varphi)$

$= -D_\lambda t \cdot D_t(\mathrm{cov}(\Delta g_P, T_Q))/(\gamma \cdot r \cdot \cos\varphi)$

(48) $\mathrm{cov}(\xi_P, \xi_Q) = -D_{\varphi'}(\mathrm{cov}(\xi_P, \zeta_Q))/r'$

$= (D^2_{\varphi\varphi'} t \cdot D_t K + D_\varphi t \cdot D_{\varphi'} t\, D^2_t K)/(\gamma \cdot r \cdot \gamma' \cdot r')$

(49) $\mathrm{cov}(\xi_P, \eta_Q) = -D_\varphi(\mathrm{cov}(\xi_P, \eta_Q))/r = (D_{\lambda'} D_\varphi t \cdot D_t K + D_\varphi t \cdot D_{\lambda'} t \cdot D^2_t K)$

$/(\cos\varphi' \cdot \gamma' \cdot r' \cdot \gamma \cdot r)$ and

(50) $\mathrm{cov}(\eta_P, \eta_Q) = -D_\lambda(\mathrm{cov}(\zeta_P, \eta_Q))/(r \cdot \cos\varphi)$

$= (D^2_{\lambda\lambda'} t \cdot D_t K + D_\lambda t \cdot D_{\lambda'} t \cdot D^2_t K)/(r \cdot \gamma \cdot \cos\varphi \cdot r' \cdot \gamma' \cdot \cos\varphi')$

Applying these equations on cov, (T_P, T_Q), (39) shows, that the quantities we need to determine are (apart from the derivatives of t with respect to the latitude and longitude):

(51) $D_t K = D_t(\mathrm{cov}^{\mathrm{I}}_k(T_P, T_Q)) + D_t(\mathrm{cov}^{\mathrm{U}}_k(T_P, T_Q))$,

(52) $D^2_t K = D^2_t(\mathrm{cov}^{\mathrm{I}}_k(T_P, T_Q)) + D^2_t(\mathrm{cov}^{\mathrm{U}}_k(T_P, T_Q))$ and

(53) $D_t(\mathrm{cov}(\Delta g_P, T_Q)) = D_t(\mathrm{cov}^{\mathrm{I}}_k(\Delta g_P, T_Q)) + D_t(\mathrm{cov}^{\mathrm{U}}_k(\Delta g_P, T_Q))$

The last term in each of the equations (51)-(53) are identical to Tscherning and Rapp (1974, eq. (108)-(110), (118)-(120) and (133)-(135)) for $k = 1, 2, 3$.

For the first term in each of the three equations we have, using (41), (42) and (43):

(54) $D_t(\mathrm{cov}^{\mathrm{I}}_k(T_P, T_Q)) = \sum_{\ell=0}^{\mathrm{I}} \epsilon_{k,\ell}(T, T)\, s^{\ell+1} P'_\ell(t)$

(55) $D_t^2(\text{cov}_k^I(T_P, T_Q)) = \sum_{\ell=0}^{I} \epsilon_{k,\ell}(T, T)s^{\ell+1}P''(t)$ and

(56) $D_t(\text{cov}_k^I(T_P, \Delta g_Q)) = \frac{R}{r}\sum_{\ell=0}^{I} \epsilon_{k,\ell}(\Delta g, T)s^{\ell+1} P_\ell'(t)$,

with $P_\ell'(t) = D_t P_\ell(t)$ and $P_\ell''(t) = D_t^2 P_\ell(t)$.

The sums (54) - (56) are evaluated in the program (subroutine PRED) using the recu sion algorithm given in Tscherning and Rapp (1974, section 9).

To avoid numerical problems for P near to Q, the following expressions are used for the evaluation of t and the derivatives of t with respect to φ and φ': (denoting: $d\lambda = \lambda - \lambda'$ and $d\varphi = \varphi - \varphi'$):

(57) $t = \cos\psi = \sin\varphi \cdot \sin\varphi' + \cos\varphi \cdot \cos\varphi' \cdot \cos(d\lambda)$

$= \cos(d\varphi) - \cos\varphi \cdot \cos\varphi' \cdot (1 - \cos(d\lambda))$

$= 1 - 2(\sin^2(d\varphi/2) + \cos\varphi \cdot \cos\varphi' \cdot \sin^2(d\lambda/2))$

(58) $D_\varphi t = \cos\varphi \cdot \sin\varphi' - \sin\varphi \cdot \cos\varphi' \cos(d\lambda) = -\sin(d\varphi) + 2\cos\varphi' \cdot \sin\varphi \cdot \sin^2(d\lambda/2)$,

(59) $D_{\varphi'} t = \sin(d\varphi) + 2\cos\varphi \cdot \sin\varphi' \sin^2(d\lambda/2)$ and

(60) $D^2_{\varphi\varphi'} t = \cos\varphi \cdot \cos\varphi' + \sin\varphi \cdot \sin\varphi' \cdot \cos(d\lambda) = \cos(d\varphi) - (1 - \cos(d\lambda)) \cdot \sin\varphi \cdot \sin\varphi'$

$= \cos(d\varphi) - 2\sin^2(d\lambda/2) \cdot \sin\varphi \cdot \sin\varphi'$.

r

We will now introduce a compact notation for the normalized surface harmonics, which will facilitate the presentation of the covariances between the coefficients of T developed in spherical harmonics and other quantities. Denoting

$$(61)\quad V_{\ell m}(\theta,\lambda)=\begin{cases}\overline{P}_{\ell|m|}(\cos\theta)\sin m\lambda, & m>0\\ \overline{P}_{\ell|m|}(\cos\theta)\cos m\lambda, & m\le 0\end{cases}$$

and the coefficients of T developed in spherical harmonics by $v_{\ell m}$ we have

$$(62)\quad v_{\ell m}=\frac{1}{4\pi}\int_{\omega} T(P)\cdot\frac{a^{\ell}}{r^{\ell+1}}\cdot V_{\ell m}(\theta,\lambda)d\omega$$

where $r=a$ in this integration and P is on the surface of the sphere of radius a. From equations (28) we get

$$(63)\quad v_{\ell m}=a*kM\cdot\begin{cases}\overline{C}_{\ell m}+J_{\ell m}\cdot(2\ell+1)^{\frac{1}{2}} & \text{for } m=0 \text{ and } \ell \text{ even},\\ \overline{S}_{\ell m} & \text{for } m>0,\\ C_{\ell|m|} & \text{for } m<0, \text{ and } m=0 \text{ and } \ell \text{ uneven}\end{cases}$$

We will now compute the covariances $cov_k(v_{\ell m}, s_j)$ where s_j is either v_{ij}, ξ_Q, η_Q, Δg_Q, or ζ_Q. These covariances are not explicitly used in the program, so we will not distinguish between the different covariance models, but denote the degree-variances by $\sigma_\ell(T,T)$, $\sigma_\ell(T,\Delta g)$ and $\sigma_\ell(\Delta g, \Delta g)$.

From eq. (62) and (33) we get

$$(64)\quad cov(T_P, v_{\ell m})=\frac{1}{4\pi}\int_{\omega} cov(T_P,T_Q)\cdot V_{\ell m}(\theta',\lambda')\frac{1}{a}\cdot d\omega$$

$$=\frac{1}{4\pi}\int_{\omega}\sum_{i=0}^{\infty}\sigma_i(T,T)s^{i+1}P_i(t)\cdot V_{\ell m}(\theta',\lambda')\frac{1}{a}d\omega$$

Using the well known summation formulae, (PG(~-81'))

$$(65)\quad P_i(t)=\frac{1}{2i+1}\sum_{j=-i}^{i}V_{ij}(\theta,\lambda)\cdot V_{ij}(\theta',\lambda'),$$

and the orthogonality property of the surface harmonics, we get

$$(66)\quad \operatorname{cov}(T_P, v_{\ell m}) = \frac{1}{4\pi}\int_{\omega}\Big(\sum_{i=0}^{\infty}\sigma_i(T,T)\Big(\frac{R^2}{r\cdot a}\Big)^{i+1}\frac{1}{2i+1}\sum_{j=-i}^{i}V_{ij}(\theta,\lambda)\cdot V_{ij}(\theta',\lambda')\,\cdot$$

$$\cdot V\ \ (\theta', A')\ \frac{1}{\mathbf{,}}\,d\omega$$

$$= \sigma_\ell(T,T)\cdot\Big(\frac{R^2}{r\cdot a}\Big)^{\ell+1}\cdot\frac{a}{2\ell+1}\cdot V_{\ell m}(\theta,\lambda).$$

Again using (62) and the orthogonality property we see, that

$$(67)\quad \operatorname{cov}(v_{ij},\ v_{\ell m}) = \frac{1}{4\pi}\int_{\omega}\Big(\sigma_\ell(T,T)\Big(\frac{R^2}{r\cdot a}\Big)^{\ell+1}\frac{a}{2\ell+1}\,V_{\ell m}(\theta,\lambda)\Big)\,\frac{a^i}{(r')^{i+1}}V_{ij}(\theta',\lambda')\Big)\,d\omega$$

$$= \begin{cases}\sigma_\ell(T,T)\Big(\dfrac{R^2}{a^2}\Big)^{\ell+1}\cdot\dfrac{}{2\ell+1} & \text{for } i=\ell \text{ and } j=m\\ 0 & \text{otherwise.}\end{cases}$$

Thus, the covariance of two different anomalous potential coefficients is zero and their variance is equal to the degree-variance multiplied by a constant depending on R, a and the degree ℓ.

The other covariance functions can be derived using (66) and (20)-(23):

$$(68)\quad \operatorname{cov}(\zeta_P, v_{\ell m}) = \sigma_\ell(T,T)\cdot a\cdot\Big(\frac{R^2}{a\cdot r}\Big)^{\ell+1}V_{\ell m}(\theta,\lambda)/(\gamma\cdot(2\ell+1)),$$

$$(69)\quad \operatorname{cov}(\xi_P, v_{\ell m}) = -\sigma_\ell(T,T)\cdot a\cdot\Big(\frac{R^2}{a\cdot r}\Big)^{\ell+1}D_\varphi V_{\ell m}(\theta,\lambda)/(r\cdot\gamma\cdot(2\ell+1))$$

$$(70)\quad \operatorname{cov}(\eta_P, v_{\ell m}) = -\sigma_\ell(T,T)\cdot a\cdot\Big(\frac{R^2}{a\cdot r}\Big)^{\ell+1}D_\lambda V_{\ell m}(\theta,\lambda)/(r\cdot\gamma\cdot\cos\varphi\cdot(2\ell+1))$$

and

$$(71)\quad \operatorname{cov}(\Delta g_P, v_{\ell m}) = \sigma_\ell(\Delta g, T)\cdot\frac{a}{R}\Big(\frac{R^2}{a\cdot r}\Big)^{\ell+1}\frac{R}{r}\,V_{\ell m}(\theta,\lambda)/(2\ell+1)$$

$$= \sigma_\ell(T,T)(\ell-1)\,\frac{a}{r}\Big(\frac{R^2}{a\cdot r}\Big)^{\ell+1}V_{\ell m}(\theta,\lambda)/(2\ell+1).$$

2.3 Stcpwise collocation.

The solution of the normal equations (8) and (9) may be a difficult numerical task even when using a large computer, when the number of unknowns is greater than a few thousands.

Moritz (1973) uses the term sequential collocation when the observations are divided in two or more groups and when the corresponding normal equations then are solved by inverting only the submatrices containing the covariances between the observations within one group.

Let us first consider an example, where the observations have been divided in two groups containing m, and m, observations respectively:

$$(72) \quad x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} ,$$

where x_1 is a m, vector and x_2 a m, vector of observations. The covariance matrix is then divided in four submatrices accordingly:

$$(73) \quad \bar{C} = \begin{Bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{Bmatrix}$$

and the vector of the covariances between the quantity s to be predicted and the observations becomes

$$(74) \quad C_s^T = \{C_{s1}^T ,\ C_{s2}^T\}.$$

Hence, according to Moritz (1973, eq. (1-22)) we have

$$(75) \quad \tilde{s} = C_{s1}^T C_{11}^{-1} x_1 + (C_{s2} - C_{s1}^T C_{11}^{-1} C_{12})(C_{22} - C_{21} C_{11}^{-1} C_{12})^{-1} (x_2 - C_{21} C_{11}^{-1} x_1)$$

Let us regard the case where we want to estimate T(P). Denoting

$$(76) \quad b_1 = C_{11}^{-1} x_1$$

$$(77) \quad \tilde{T}_1(P) = C_{P1}^T \cdot b_1$$

$$(78) \quad d_1 x_2 = x_2 - C_{21} C_{11}^{-1} x_1 = x_2 - C_{21} \cdot b_1$$

(79) $d_1 C_{22} = C_{22} - C_{21} C_{11}^{-1} C_{12}$, $d_1 C_{P2} = C_{P2} - C_{P1} C_{11}^{-1} C_{12}$

(80) $b_2 = d_1 C_{22}^{-1} d_1 x_2$ and

(81) $\tilde{T}_2(P) = d_1 C_{P2}^T \cdot b_2$,

we see that

$$\tilde{T}(P) = \tilde{T}_1(P) + \tilde{T}_2(P)$$

Hence, we are by using stepwise collocation, getting an estimate $\tilde{T}$ which is equal to the sum of two other estimates.

The first estimate $\tilde{T}_1(P)$ is computed by (76) and (77) from the observations x_1 using the original covariance functions. The residual observations $d_1 x_2$ (78) is then computed. Then the second estimate $\tilde{T}_2(P)$ can be obtained from (80) and (81) using the covariances of the residual observations, $d_1 C_{22}$ and $d_1 C_{P2}$ (79). (Supposing the "noise" matrix D to be zero we easily see

$$\begin{aligned} \mathrm{cov}(d_1 x_{2i}, d_1 x_{2j}) &= \mathrm{cov}(x_{2i} - \{\mathrm{cov}(x_{2i}, x_{1j})\}^T \{\mathrm{cov}(x_{1j}, x_{1k})\}^{-1} \{x_{1k}\}, \\ &\qquad x_{2j} - \{\mathrm{cov}(x_{2j}, x_{1\ell})\}^T \{\mathrm{cov}(x_{1\ell}, x_{1k})\}^{-1} \{x_{1k}\}) \\ &= \mathrm{cov}(x_{2i}, x_{2j}) - \{\mathrm{cov}(x_{2i}, x_{1j})\}^T \{\mathrm{cov}(x_{1j}, x_{1k}\}^{-1} \{\mathrm{cov}(x_{1k}, x_{2j})\} \end{aligned}$$

which is nothing but the i, jth element of the matrix $d_1 C_{22}$.)

The formulae (76)-(81) can be generalized as to describe a partition of the observations into more than two groups. Such equations can for example, be found in Moritz (1973, eq. (5-1)-(5-14)). We will here use a slightly different type of general equation.

For a partition in k groups,

$$x = \begin{Bmatrix} x_1 \\ \vdots \\ x_k \end{Bmatrix}, \quad x_i = \begin{Bmatrix} x_{i1} \\ \vdots \\ x_{im_i} \end{Bmatrix}$$

and with

$$(78a)\quad \begin{cases} d_j x_i = x_i - \sum_{\ell=1}^{j} d_{\ell-1} C_{i\ell}^T \cdot b_\ell \\ d_j T_P = T_P - \sum_{\ell=1}^{j} d_{\ell-1} C_{P\ell}^T \cdot b_\ell \\ d_j s = s - \sum_{\ell=1}^{j} d_{\ell-1} C_{s\ell}^T \cdot b_\ell \quad , \end{cases}$$

$$(79a)\quad \begin{cases} d_\ell C_{ij} = \{\mathrm{cov}(d_\ell x_{ik},\ d_\ell x_{jn})\} & k = 1, \ldots, m_i \\ & n = 1, \ldots, m_j \\ d_\ell C_{Pi} = \{\mathrm{cov}(d_\ell T_P,\ d_\ell x_{ij})\} & j = 1, \ldots, m_i \\ d_\ell C_{si} = \{\mathrm{cov}(d_\ell s,\ d_\ell x_{ij})\} & j = 1, \ldots, m_i \end{cases}$$

and

$$(80a)\quad b_i = d_{i-1} C_{ii}^{-1} \cdot d_{i-1} x_i$$

we have the estimates $\tilde{T}_i(P)$ and $\tilde{s}_i$ based on the residual observations $d_{i-1} x_i$ (i.e. on all sets of observations with subscript less than i):

$$(81a)\quad \begin{cases} \tilde{T}_i(P) = d_{i-1} C_{Pi}^T \cdot b_i \\ \tilde{s}_i = d_{i-1} C_{si}^T \cdot b_i \end{cases}$$

and the final estimates

$$(82a)\quad \begin{cases} \tilde{T}(P) = \sum_{i=1}^{k} \tilde{T}_i(P) \ , \\ \tilde{s} = \sum_{i=1}^{k} \tilde{s}_i \ . \end{cases}$$

(As usual, the linear equations corresponding to (80a) are denoted the normal-equations and b_i the solution to the normal equations.)

One of the main advantages achieved by using stepwise collocation is according to Moritz (1973, page 1), that the normal equation matrices to be inverted are smaller than the original $\overline{C}$ matrix. Thus, as may be realized from equations (79) and (80), the total storage requirements are not diminished. So, when using a computer, which has peripheral storage units with fast access, stepwise solution of the normal equations is of no real advantage.

Thus, a considerable simplification of the computations may be achieved when the residual covariances $d_\ell C_{ij}$ and $d_\ell C_{Pi}$ (79a) can be computed analytically. In this case, only the b_i vectors (80a) are needed for the representation of $\widetilde{T}_i$ and for the computation of p edictions. The residual covariance may naturally be computed analytically, when the datasets are uncorrelated, i.e. when $d_\ell C_{11} = C_{11}$. But the possibility for analytical covariance also exists, when the first dataset x_1 consists of potential coefficients.

The matrix C_{11} is in this case a diagonal matrix with diagonal elements equal to the sum of the quantities given in (67) and the error variances of the observed coefficient, $\sigma^2_{v_{\ell m}}$. We will suppose, that all the variances are the same for the same degree, R, and denote this variance by $\sigma_v^2(\ell)$.

The elements of the vector b_1 are equal to the observed coefficient divided by the corresponding diagonal element. Let us then suppose, that potential coefficients up to degree I have been observed. The estimate $\widetilde{T}_1$ is then (cf. eq. (66), (67) and (77)):

$$\text{(83a)}\quad \widetilde{T}_1(P) = \sum_{\ell=2}^{I} \sum_{m=-\ell}^{\ell} a \cdot \left(\frac{R^2}{r \cdot a}\right)^{\ell+1} \sigma_\ell(T,T) \cdot V_{\ell m}(\theta,\lambda) \frac{1}{2\ell+1} \cdot$$

$$r(v_{\ell m} / (a^2 \left(\frac{R}{a}\right)^{2\ell+2} \cdot \frac{\sigma_\ell(T,T)}{2\ell+1} + \sigma_v^2(\ell)))$$

Denoting $\widetilde{v}_{\ell m} = v_{\ell m}/(1 + \sigma_v^2(\ell) \left(\frac{a}{R}\right)^{2\ell+2} \frac{2\ell+1}{a^2 \cdot \sigma_\ell(T,T)}$) we have

$$\text{(83b)}\quad \widetilde{T}_1(P) = \sum_{\ell=2}^{I} \frac{a^\ell}{r^{\ell+1}} \sum_{m=-\ell}^{\ell} \widetilde{v}_{\ell m} \cdot V_{\ell m}(\theta,\lambda).$$

Using equations (66), (67)-(79) and (85b) we easily see, that the residual observations (78), d_1x_2 are nothing but the original observations, but now referring to a higher order reference field, $U_1 = U + T_1$.

For the i, j'th element of d_1C_{22} we get from (GG) and (79), supposing e.g. that $s_{2i} = T(P)$, $s_{2j} = T(Q)$ and that x_{2i}, x_{2j} are the corresponding observed values (and P different from Q):

$$(84)\quad \operatorname{cov}(d_1 s_{2i}, d_1 s_{2j}) = \sum_{\ell=2}^{\infty} \sigma_\ell(T,T)\left(\frac{R^2}{r \cdot r'}\right)^{\ell+1} P_\ell(t) - \sum_{\ell=2}^{I}\sum_{m=-\ell}^{\ell}\left(a\left(\frac{R^2}{a \cdot r}\right)^{\ell+1}\right.$$

$$\cdot \frac{\sigma_\ell(T,T)}{2\ell+1} \cdot V_{\ell m}(\theta,\lambda)\Big)\left(a\left(\frac{R^2}{a \cdot r'}\right)^{\ell+1} \cdot \frac{\sigma_\ell(T,T)}{2\ell+1} \cdot V_{\ell m}(\theta',\lambda')\right)\Big/\Big(a^2$$

$$\left.\cdot \left(\frac{R}{a}\right)^{2\ell+2} \frac{\sigma_\ell(T,T)}{2\ell+1} + \sigma_v^2(\ell)\right)$$

$$= \sum_{\ell=2}^{I} d_1\sigma_\ell(T,T) s^{\ell+1} P_\ell(t) + \sum_{\ell=I+1}^{\infty} \sigma_\ell(T,T) s^{\ell+1} P_\ell(t),$$

with

$$(85)\quad d_1\sigma_\ell(T,T) = \sigma_\ell(T,T)\left(1 - 1\Big/\left(1 + \sigma_v^2(\ell)\Big/\left(a^2\left(\frac{R}{a}\right)^{2\ell+2}\frac{\sigma_\ell(T,T)}{2\ell+1}\right)\right)\right).$$

This quantity is zero for $\sigma_v^2(\ell)$ equal to zero. The covariance function (84) is in this case a local I'th order covariance function, cf. Tscherning and Rapp (1974, Section 9).

It is supposed in the program, that the error variances $\sigma_{v_{\ell m}}^2$ can be either disregarded or that they only depend on the degree. The program will, in the latter case, require that the quantity

$$(86)\quad d_1\sigma_\ell(Q,\Delta g)= d_1\sigma_\ell(T,T)\cdot\frac{(\ell-1)^2}{R^2}$$

is specified. The quantity will be treated as if it was an empirical degree-variance.

We have here seen, how we in one case explicitly can derive expressions for the covariance, $\mathrm{cov}(d_1x_{2i}, d_1x_{2j})$. Anolher method would he simply to estimate a covariance function for the rcsidual. observations $d_1 x_2$. However, the program can only use three types of covariance functions, and they are all isotropic.

We are then restricted either to divide the observations x in groups of quantities, which are nearly uncorrelated or to find a kind of observations, which we can treat like the potential coefficients.

The potential coefficients are (in a general sense) weighted mean values of the anomalous potential. Mean gravity anomalies are also mean values, weighted with a function, which is equal to one in the considered area and zero outside.

A mean gravity anomaly field will represent an amount of information which is equal to the amount of information contained in a set of potential coefficients of degree less than or equal to an integer I. The magnitude of I will depend on the size of the area over which the mean anomaly is computed. I will be large when the area is small and small when the area is large. (I will be zero when the area is the whole Earth and infinite when we are dealing with points). An estimate of the degree may be found in the following way: The total number of equal area mean anomalies of a particula size is theoretically equal to the total area of the Earth divided by the area of the basic mean anomaly. Let us call this number N. For the perfect recovery of N quantities we need a set of coefficients of degree up to $N^{\frac{1}{2}}$. This method of estimation will give us $N \approx 202$ for 1° equal area anomalies. The degree may also be estimated in a more empirical way. This can be done by first estimating the empirical covariance function of a set of residual observations (gravity anomzlies) $d_2 x_3$. The first zero point of the empirical. covariance function (regarded as a function of the spherical distance ψ) will then give a reasonable estimate of the degree (cf. Tscherning and Rapp, (1974, Section 9)).

As an example, the program described in this report was used to compute residual point gravity anomalies in a $2^\circ 30' \times 3^\circ 40'$ square in the state of Ohio, U. S.A. The data set x did consist of three groups. The set x_1 was a set of potential coefficients of degree upto and inclusive of 20, given by Rapp (1973, Table G). x_2 consisted of 157 $1^\circ \times 1^\circ$ mean gravity anomalies surrounding the area and x_3 was a set of 420 point gravity anomalies, spaced as uniform as possible with a distance of $7\frac{1}{2}'$ in latitude and $10'$ in longitude between the points. The data-sets x_1 and x_2 was regarded as errorless.

The covariance function recommended by Tscherning and Rapp (1974) was used (i.e. given by eq. (32) with B = 24 and A, = 425 mgal^2). The covariances $d_1C_{s_i s_j}$ can then be computed using a corresponding local 20'th order covariance function (i.e. with degree-variances of order up to and inclusive of 20 equal to zero), cf. (84).

The function $\tilde{T}_1$ is then computed without actually solving any normal equations. We have (cf. (83a) and with $\sigma_v^2(\ell)=0$):

$$T_1 = C_{P1}^T \cdot b_1 = \sum_{\ell=2}^{1} \sum_{m=-\ell}^{\ell} a \cdot \left(\frac{R^2}{r \cdot a}\right)^{\ell+1} \sigma_\ell(T,T) \cdot V_{\ell m}(\theta,\lambda) \frac{1}{2\ell+1} \cdot b_{1\ell m} ,$$

$$b_{1\ell m} = v_{\ell m} / (a^2 \left(\frac{R}{a}\right)^{2\ell+2} \cdot \frac{\sigma_\ell(T,T)}{2\ell+1}) .$$

We will now, as mentioned above represent the mean gravity anomaly as a point anomaly in a certain height h' above the center of the area. Let us denote this point by Q and its distance from the origin r'. (We have in this case used $h' = r' - R$, = 10.5 km, cf. Tscherning and Rapp (1974, Section 10)).

The residual anomaly is then, cf. eq. (78a) and (71):

$$d_1 x_{2j} = x_{2j} - \sum_{\ell=2}^{1} \sum_{m=-\ell}^{\ell} \operatorname{cov}(x_{2j}, v_{\ell m})\, b_{1\ell m}$$

$$= x_{2j} - \sum_{\ell=2}^{1} \sum_{m=-\ell}^{\ell} \sigma_\ell(T,T)(\ell-1) \frac{a}{r'} \left(\frac{R^2}{a \cdot r'}\right)^{\ell+1} V_{\ell m}(\theta',\lambda') \frac{1}{2\ell+1} \cdot b_{1\ell m}$$

$$= x_{2j} - \sum_{\ell=2}^{1} \sum_{m=-\ell}^{\ell} v_{\ell m} \cdot (\ell-1)\left(\frac{a}{r'}\right)^{\ell+1} \frac{1}{r'} V_{\ell m}(\theta',\lambda')$$

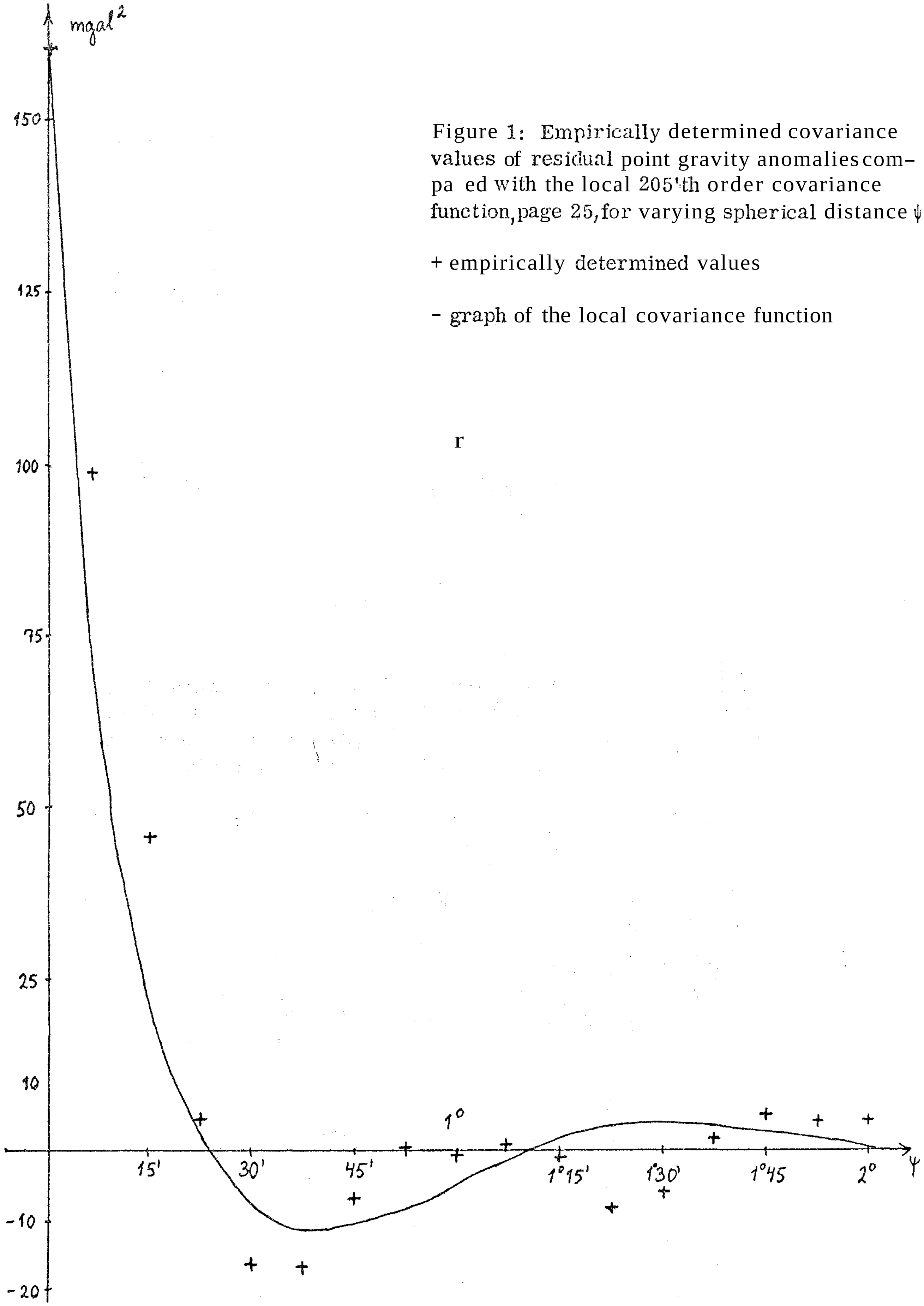


Figure 1: Empirically determined covariance values of residual point gravity anomalies compa ed with the local 205'th order covariance function, page 25, for varying spherical distance ψ.

+ empirically determined values

- graph of the local covariance function

$$= x_{2j} - \left(-\frac{\partial}{\partial r'}\tilde{T}_1(Q) - \frac{2}{r'}\tilde{T}_1(Q)\right),$$

i.e., the mean anomaly computed with respect to the higher order reference field $U_T = Us\ \tilde{T}_1$. (The program does not use this equation for the computation of the residual anomaly. The contribution from $\tilde{T}_1$ is evaluated using the actual length of the gradient of U_1, see the description of the subroutine IGPOT, Section 5.4).

The residual observations d_1x_2 was then used to determine $\tilde{T}_2$, this time by actually solving a set of normal equations, obtaining the solution vector b_2.

The residual point gravity anomalies were then computed by

$$d_2 x_3 = x_3 - d_1 C_{23}^T \cdot b_2 - C_{13}^T \cdot b_1$$

cf. (78a). The term $C_{13}^T \cdot b_1$ is again here the change due to the higher order reference field U_1.

The empirical covariance function was computed using $d_2 x_3$ by taking the sample mean of the products of the residuals sampled according to the spherical distance between the points of observation. The size of the sample interval was $7\frac{1}{2}'$. The covariance function

$$\mathrm{cov}(\Delta g_P, \Delta g_Q) = \sum_{\ell=2.05}^{\infty} \frac{81.8\ \mathrm{mgal}^2 \cdot (\ell-1)}{(\ell-2)(\ell+24)} \cdot s^{\ell+2} P_\ell(\cos\psi),$$

with $R/R_e = 0.9998$ was found to have the same zero point as the empirical covariance function, see Figure 1.

We then see, that the two mentioned methods give nearly the same estimate of the integer L This agreement should merely be taken as an illustration and not as a proof. It shows one of the many kinds of coinputaticns the FORTRAN program can perform.

The choice of a proper covariance function is a delicate task, but we point out that the presented program may use three different degree-variance models and hence be useful in test computations using different covariance functions.

The program may compute $\tilde{T}$ in up to three steps. The different possibilities are illustrated in Table 1. Note, that potential coefficients always form a separate data set, which will be the data set x_1.

Table 1

Different: options for the Computation of $\tilde{T}$.

Number of steps	Dataset May Contain: x_1	x_2	x_3	$\tilde{T} =$
1	Potential coefficients			$\tilde{T}_1$
1	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$			$\tilde{T}_1$
2	Potential coefficients	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$		$\tilde{T}_1 + \tilde{T}_2$
2	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$		$\tilde{T}_1 + \tilde{T}_2$
3	Potential coefficients	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$	$\xi, \eta, \zeta, \Delta g, \overline{\Delta g}$	$\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_3$

3. Data Requirements.

In this section the data requirements will be discussed. The precise specifications are given in Section 6.

Three types of information are needed for the determination of T: observations, information about the reference system of the observations and a covariance function.

3.1 The Observed Quantities.

The observed quantities we want to use are (a) potential coefficients, (b) point or mean free air gravity anomalies or measured gravity values, (c) height anomalies and (d) deflections of the vertical.

The potential coefficients available will generally all be of a degree less than 25. There has then only been reserved space in core store for up to 625 coeflicients.

The program accepts potential coefficients, which are fully normalized and multiplied by 10^6. The coefficients can naturally only be used, when a value of kM and the semi major axis a are specified.

An observation (different from a potential coefficient) will be given by (1) the geodetic latitude and longitude, (2) a potential difference, (a geopotential number, for example), conve ted into a metric quantity e. g. by dividing the difference with the reference gravity and (3) the measured quantity. The height above the reference ellipsoid is regarded as unknown except, naturally, when it itself is the observed quantity.

A11 measured gravity values will have to be given in the same gravity reference system or a correction must be known. Measured gravity values are converted to free-air anomalies. The orthometric height must hence be known. The geodetic latitude (which is used to evaluate the normal gravity) would principally have to be given in a geodetic reference system consistent with the gravity reference system (i.e. with the same flattening and semi major axis as used for the computation of the coefficicnts in the expression for the normal gravity). But the variation of the normal gravity with respect to the latitude is so small, that this requirement can be neglected here. The point or mean gravity anomalies will all have to be free-air anomalies. They must all refer to the same normal gravity field. If they are not all given with respect to the same gravity base reference system, the correction to be applied for the conversion must be known.

A mean gravity anoinaly will be represented as a point gravity anomaly at a point of a certain height, h, above the center of the area over which the mean value is computed. This height is specified by the ratio, RP between the sum of this height and the mean Earth radius R_e and the mean Earth radius, i.e.

(87) $RP = (R_e + h)/R_e$.

A height anomaly will have to be given in the same reference system as the geodetic latitude and longitude. This will generally require, that a height anomaly obtained through an absolute position determination and given in a geocentric reference system must be transferred back to a local geodetic refe ence system, before it can be used in the program.

We are, with observed deflections of the vertical, faced with a complicated problem. The deflections are equal to the difference between the astronomical coordinates of a point on the geoid and the geodetic coordinates of a point on the reference ellipsoid (multiplied with cosine to the latitude for the longitude difference).

We have hereby implicitly introduced assumptions about the mass densities in between the geoid and the astronomical station. To avoid this, the deflections should have been given at the proper height (i.e. the height of the observation stations).

Thus, heights of astronomical stations are seldom found recorded together with the deflections. But if the heights are actually recorded, the program will treat the deflections as quantities, which have not been reduced to the geoid.

The astronomical coordinates may carry systematical errors due to systematic differences between star catalogues or due to the neglect of corrections for polar motion. The observations may be corrected for known systematical errors, if they can be specified in the same way as a datum shift, i. e. by specifying the corrections in the latitude and the longitude components at a certain point. Systematic errors in the height anornalies may be corrected in the same manner.

In Section 2.2 we mentioned, that the equations which related the observations and the anomalous potential was given in spherical approximation. This means, e.g. that all points on the surface of the Earth are regarded as lying on the mean Earth sphere. This fact has been used in the program to speed up the computations. This is done by using the fact that the quantities

$$\sigma_\ell(T, T)\, s^{\ell+1}, \quad \sigma_\ell(T, \Delta g)\, s^{\ell+1}, \quad \text{and } \sigma_\ell(\Delta g, \Delta g)\, s^{\ell+2}$$

will be the same for a group of input data.

Data which actually are observed above the surface of the Earth must be grouped so that they all refer to a sphere with radius equal to a mean height of the points plus the mean Earth radius, R,. As for mean gravity anomalies, the height is specified in the program through the value of the quantity RP, (ey. (87)).

The standard deviations of observations, different from potential coefficients, will have to be given in meters for the height anomaly, in mgal for gravity observations and in arc sec for deflection components. The standard deviations may be specified (1) individually for the single observations, (2) for a group of observations or (3) as being zero for all observations.

3.2 The Reference Systems.

We have to know the parameters specifying the geodetic coordinate system. The program requires the semi major axis, a, and the flattening, f, to be specified.

The gravity formulae may then either be given (1) through the values of kM, ω, a, and f, (2) by specifying that the international gravity formulae and the Potsdam reference system has been used or (3) that the Geodetic Reference System 1967 has been used. One of the three excludes the others.

The covariance functions which can be used, will all have the degree-variances of degree zero and one equal to zero. This implies, that we, in the computations, have to use the best possible kM value and a geodetic coordinate system which has origin coinciding with the gravity center of the Earth, Z-axis parallel to the mean axis of rotation and Z-X-plane equal to the mean Greenwich meridian plane.

We will also require the global mean value of the gravity anomalies, the height anomalies and the deflections to be zero. This requirement implies, that we have to use the best possible semi-major axis. The geodetic latitude and longitude may then be transformed into such a reference system by specifying the new kM, a, f values, the translation vector, the scale change and the three rotation angles for the rotations around the X, Y and Z axes respectively.

The approximation $\tilde{T}$ will be given in the same reference system as the one specified through the transformation parameters, i. e. in a geocentric reference system. Predictions will be given in both the original and the new reference system.

3.3 The Covariance Function.

We explained in Section 2.2 how an isotropic covariance function can be specified through (1) a set of empirical anomaly degree-variances of degree less than or equal to an integer I, $\hat{\sigma}_{\ell}(\Delta g, \Delta g)$ and (2) an anomaly degree-variance model for the degree-variances $\sigma_{k,\ell}(\Delta g, \Delta g)$ for ℓ greater than I.

The values of the empirical degree-variances will depend on the radius of the Bjerhammar sphere, R. We have, therefore chosen to specify these quantities on the surface of the mean Earth, i. e. the quantities

$$(88) \quad \hat{\sigma}^{E}_{\ell}(\Delta g, \Delta g) = \left(\frac{R}{R_e}\right)^{2\ell+4} \hat{\sigma}_{\ell}(\Delta g, \Delta g)$$

must be given together with the ratio R/R_e. The quantities (88) must be given in units of $mgal^2$.

The anomaly degree-variance model is for k = 1 and 2 specified through the constants A_1 and A_2 (eq. (30) and (31)) and for k = 3 through the constant A_3 and the integer B (eq. (32)).

Thus, in the program the models are specified not through the constants A_k, k = 1, 2 or 3, but through the variance of the point gravity anomalies on the surface of the Earth.

This quantity is then used for the determination of A_k. We have from **(29):**

$$(89) \quad \mathrm{cov}(\Delta g_P, \Delta g_P) = \sum_{\ell=0}^{I} \hat{\sigma}_k(\Delta g, \Delta g) \cdot \left(\frac{R}{R_e}\right)^{2\ell+2} P_{\ell}(1)$$

$$+ \sum_{\ell=I+1}^{\infty} \sigma_{k,\ell}(\Delta g, \Delta g) \left(\frac{R}{R_e}\right)^{2\ell+2} P_{\ell}(1)$$

Let us now, for example regard model 2, i. e. $\sigma_{2,\ell}(\Delta g, \Delta g) = \dfrac{A_2(\ell-1)}{(\ell-2)}$. Then

$$(90)\quad A_2 = \left(\mathrm{cov}(\Delta g_P, \Delta g_P) - \sum_{\ell=0}^{I} \sigma_k^h(\Delta g, \Delta g)\left(\frac{R}{R_e}\right)^{2\ell+2}\right) \Big/ \left(\sum_{\ell=I+1}^{\infty} \frac{(\ell-1)}{(\ell-2)}\left(\frac{R}{R_e}\right)^{\ell+2}\right)$$

The infinite sum may be computed by the formula given in Tscherning and Rapp (1974, Section 8), and A, (and in the same way A, or A,) can then be found.

We will finally mention, that the ratio R/R_e is used by the program for the computation of the radius of the Bjerhammar sphere.

4. Main Lines of Function of the Program.

In Section 2 we mentioned that the program could be used to estimate $\tilde{T}$ from maximally three sets of observations x_1, x_2 and x_3. $\tilde{T}$ would then be equal to the sum of up to three harmonic functions, $\tilde{T}$, , $\tilde{T}_2$ and $\tilde{T}_3$. The limit of three was only attained, when potential coefficients formed the first set of observations, x_1.

We will here clescribe the function of the program, when we are in this situation, i.e. when we have three datasets x_1, x_2 and x_3, and x_1 is a set of potential coefficients.

The flow of the program is illustrated in Figure 2. Several logical variables determine the flow. The logical variable LPRED will e.g. be "false" until the estimation of? is finished and will have the value "true", when predictions are computed.

The program will start by initializing different variables. It will require information about the reference systems of the observations and use this information to select e.g. the proper formulae for the normal gravity.

When the reference system is not geocentric or when the normal gravity does not correspond to a proper kM value, the necessary transformation elements and the kM value must be given.

The next step is then to read in the observations x_1, the potential coefficients. The normal equations (12) will not have to be solved in this situation, cf. Section 2.3. $\tilde{T}_1$ is represented by (83).

The following two steps, where we explicitly use the equations for collocation, will be denoted Collocation I and Collocation II. We will first have to specify the covariance function and observations used in Collocation I:

The covariance function for the residual anomalies $d_1 x_2$ must be specified through the selection of an anomaly degree-variance model and contingently by specifying a set of low order empirical. deg ee-variances.

The observations x, (and later x,) may be subdivided in different files according to format, kind of observed quantity etc. Each single observation is first transformed to a geocentric reference system (if necessary). Then the residual observation is computed, by subtracting the contribution from $\tilde{T}_1$ from the observed value. After the input of a file, the value of a local variable LSTOP will be input, which will signify if more files belonging to x_2 will have to be input.

Figure 2

Flow-Chart of Program.

The main flow is determined by the values of the following logical variables:

LTRAN = coordinates and observations must be transformed to a geocentric reference system and gravity observations must refer to a gravity formulae consistent with the reference system.

LPOT = potential coefficients from first set of "observed" quantities.

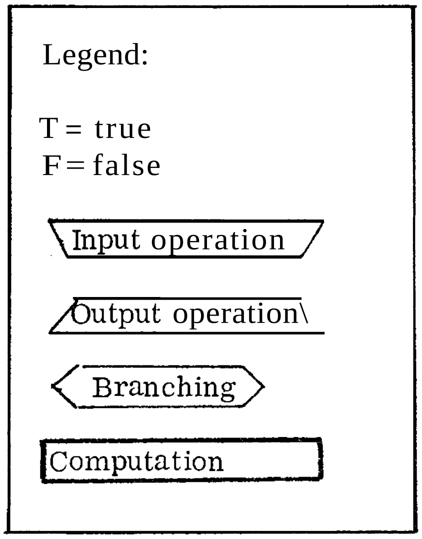
LCREF = second set of observations (or third when LPOT is true) will be used, and the harmonic function computed by Collocation I will be used as an improved reference field. LCREF is initialized to be false and will get its final value after Collocation I is finished.

LPRED = predictions are being computed.

LGRID = the predictions are computed in the points of a uniform grid.

LERNO = the error of prediction must be computed.

LCOMP = compare observed and predicted values (an observed value is input together with the coordinates of the point of prediction).



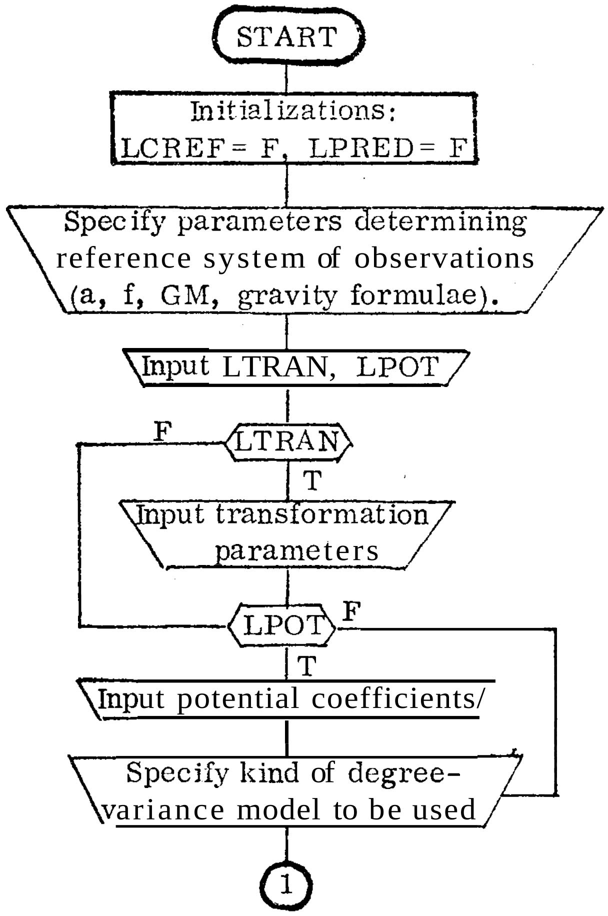
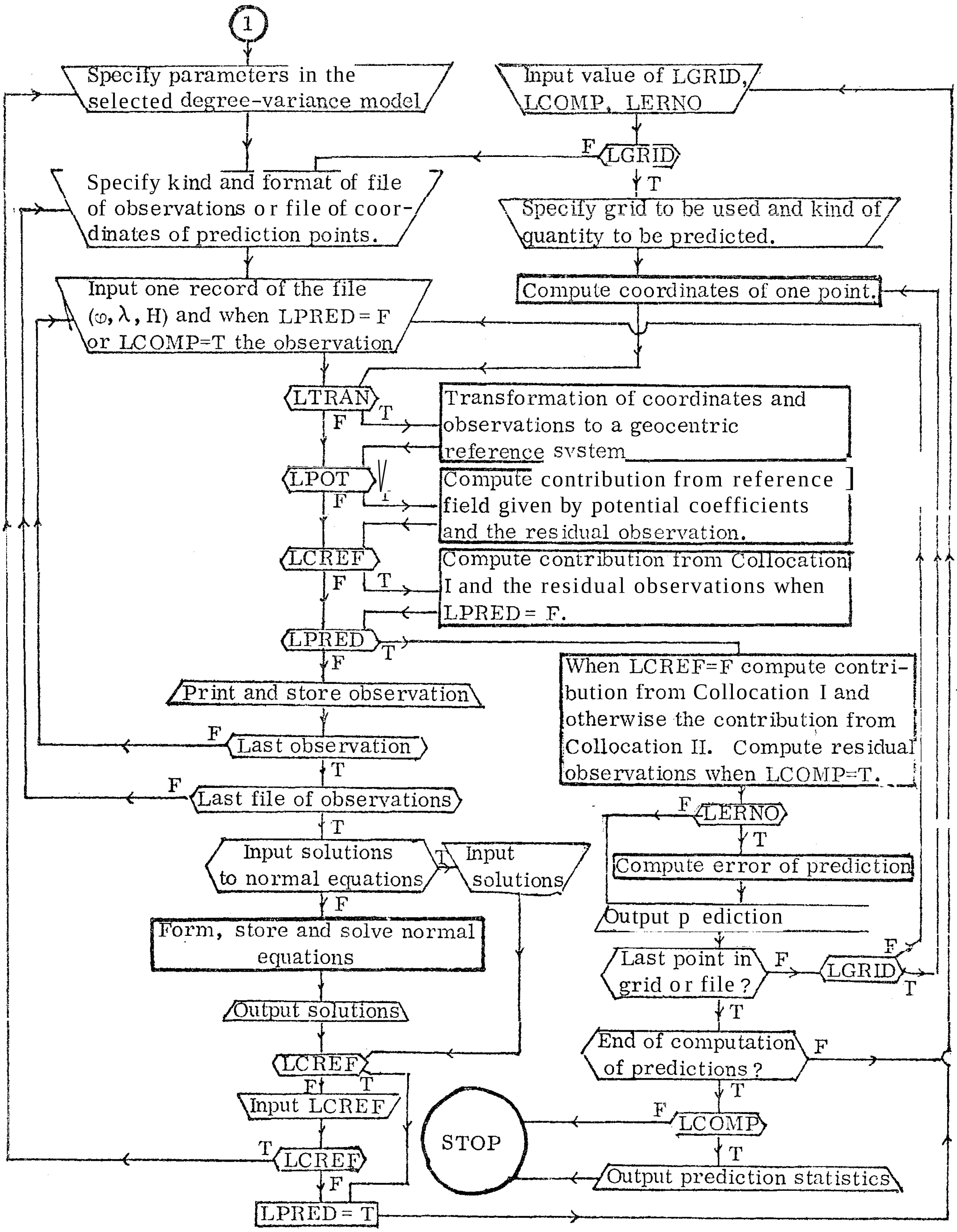


Figure 2
(cont'd)



After the last file has been input, the vector $d_1 x_2$ is stored on a disk. The coefficients of the normal equations are then computed and stored on the disk as well.

A subprogram NES, which only uses a limited amount of core store, will then compute the solution vector b_2 and in this way $\tilde{T}_2$ is determined.

The solution vector may be output on punched cards, so that the function $\tilde{T}_2$ can he retrieved without computing the coefficients of (and solving) the normal equations.

Collocation I is now finished. It is then possible either to start the computation of predictions or to start Collocation II. A logical variable LCREF is used to distinguish between the two situations. Thus, LCREF will have to be true in this case, because we have decided to describe the situation, where three datasets are used.

The covariance function of the residual observations $d_2 x_3$ is then first specified. It is done in the same way as for the covariance function used in Collocation I, though the kind of anomaly degree-variance model used will have to be the same.

The different files of the dataset x_3 can then be input. Each observations is first transformed to a geocentric reference system. Then the contribution from $\tilde{T}_1$ and $\tilde{T}_2$ is computed, so that finally $d_2 x_3$ can be stored. The coefficients of the new normal-equations can then be computed and the equations solved. Again, here the solution b_3 may be output on punched cards. (In case b_2 or b_1 had been computed in previous runs of the program, their respective values would have been input and the coefficients of the normal equations are then not computed.)

When the equations have been solved, the reduced normal equation matrix is retained on a disk, so that errors of estimation can be computed, using equation (13).

The estimate of the anomalous potential is then, cf. eq. (82a)

$$\tilde{T}(P) = \tilde{T}_1(P) + \tilde{T}_2(P) + \tilde{T}_3(P),$$

with $T_1(P)$ given by eq. (83),

$$T_2(P) = \sum_{i=1}^{n_1} \operatorname{cov}(d_1 T_P, d_1 x_{2i}) \cdot b_{1i} \quad \text{and}$$

$$T_3(P) = \sum_{i=1}^{m_2} \text{cov}(d_2 T_P, d_2 x_{3i}) \cdot b_{2i},$$

cf. eq. (81a).

The prediction of a height anomaly, a gravity anomaly or a deflection component can now be computed using eq. (82a). The computation is based upon exactly the same type of information as was used for the computation of $\tilde{T}_2$ and and $\tilde{T}_3$, i. e. geodetic latitude and longitude, and a height. The program itself may generate lists of coordinates. Such a list is generated, when the logical LGRID is true. The list will consist of coordinates of points lying in a grid. The grid is specified by its south-west corner, and the number and magnitude of the grid increments in northern and eastern direction. The heights of the points are specified by the ratio RP (equation (87)).

The prediction of a quantity, e.g. a gravity anomaly will then be computed by first determining the difference between the anomaly given in a geocentric reference system and the reference system of the observations, Δg_0. The contribution $\widetilde{\Delta g}_1$ is then evaluated from $\tilde{T}_1$ and the contributions from $\tilde{T}_2$ and $\tilde{T}_3$ using (81a), i. e.

$$\widetilde{\Delta g}_i = \sum_{j=1}^{m_i} \text{cov}(d_{i-1}\Delta g \ , \ d_{i-1}x_{ij})\cdot b_{ij}, \ i=2,3.$$

The predicted value is then, (cf. eq. (82a)):

$$\widetilde{\Delta g} = \Delta g_0 + \widetilde{\Delta g}_1 + \widetilde{\Delta g}_2 + \widetilde{\Delta g}_3 \, .$$

Predictions of other quantities are computed in the same way. A special facility for the comparison of observed and predicted quantities can be used, when the logical LCOMP is true. The differences between observed and predicted quantities are in this case, computed together with their mean value and variance. A sampling of the differences is done in intervals of a specified magnitude.

The processing time (or more correctly, the central unit processing time) will vary depending on (1)the covariance function used, (2) the number of observations and (3) the number of quantities to be predicted and estimates of errors to be computed. The program has been used for a variety of test computations, though never with more than 500 observations. The used processing times for a number of situations are presented in Table 2. The computations were all made on the IBM system/370 model 165 computer of the Instruction and Research Computer Centcr, Ohio State University. The so called Fortran H-extended compiler (IBM (1972)) was used for the compilation of the program. The normal equations were stored on an IBM model 3330 disk.

Table 2

Examples of Processing Times for Different Input Data and Covariance Functions. Potential Coefficients of Degree up to 20 Used.

	Collocation I		Collocation II**			Predictions			Total processing time	
Covariance Model	Order of local covar. fct. I	Number of $\overline{\Delta g}$ used	Order of local covar. fct. I	Number of Δg	Number of ξ,η	Number of Δg	Number of ξ,η	Number of ζ	m	sec
1	20	157				117	30	36	0	43
2	20	157				117	30	36	0	43
2	20	157	110	117		303	82	36	2	31
3*	20	157	110	117		303	82	36	3	22
2	20	157	160	117		303	82	36	2	57
3*	20	157	160	117		303	82	36	4	09
2	20	157	110	117	30	303	52	36	2	54
3*	20	157	110	117	30	303	52	36	4	09

* B = 24 in Model 3

**Normal equations not computed and solved in Collocation I.

5. The Storing of the Cocffic ients of the Normal Equations and the Function of the Subprograms.

We will in this section discuss in more detail the function of an important part of the main-program and the different subprograms which have been used. However, the most detailed description is found in the appendix, where the FORTRAN program, which includes a large number of comment statements, is reproduced.

5.1 The storing of the normal equations.

The IBM system 370 model 165 computer of the Instruction and Research Computer Center of the Ohio State University makes available a 630K !byte) core storage for a usual program. Let us suppose, that we have used 180 K for the storing of the program and variables different from the coefficients of the normal-equations. We are then left with 450K bytes, which can be used to store these quantities. When the coefficients are represented as double p ecision variables (8 bytes), it is then possible to store 450*1024/8=57600 coefficients in the core.

A system of equations with N unknowns, and a full symmetric coefficient matrix plus a constant vector of length N+1 will totally occupy (N+2)*(N+3)/2 8 byte positions. This implies, that we maximally can solve a system of equations with 336 unknowns, if we want to store all the coefficients in the core.

The solution to the problem is naturally to divide the upper (or lower) triangular part of the matrix in blocks, which then are stored on a disk and later read into core storage when needed. The subdivision in blocks can be made in several ways. In case we wanted to compute the inverse matrix, the optimal subdivision seems to be a subdivision in squares submatrices, as used by Karki (1973). It is unnecessary to compute the inverse matrix for our purpose. The solution vector b (16) and the estimate of the error of prediction (13) may both be computed without using the inverse matrix. It is enough to compute the so .called reduced matrix L^T,

$$(91) \quad \overline{C} = L \cdot L^T$$

where L is a lower triangular matrix, cf. Poder and Tscherning (1973). The computation of L^T is most easily programmed, when the upper triangular part of $\overline{C}$ is subdivided in blocks, which contain a number of consecutive columns, stored in a one-dimensioned array with the diagonal element having the highest subscript cf. Figure 3.

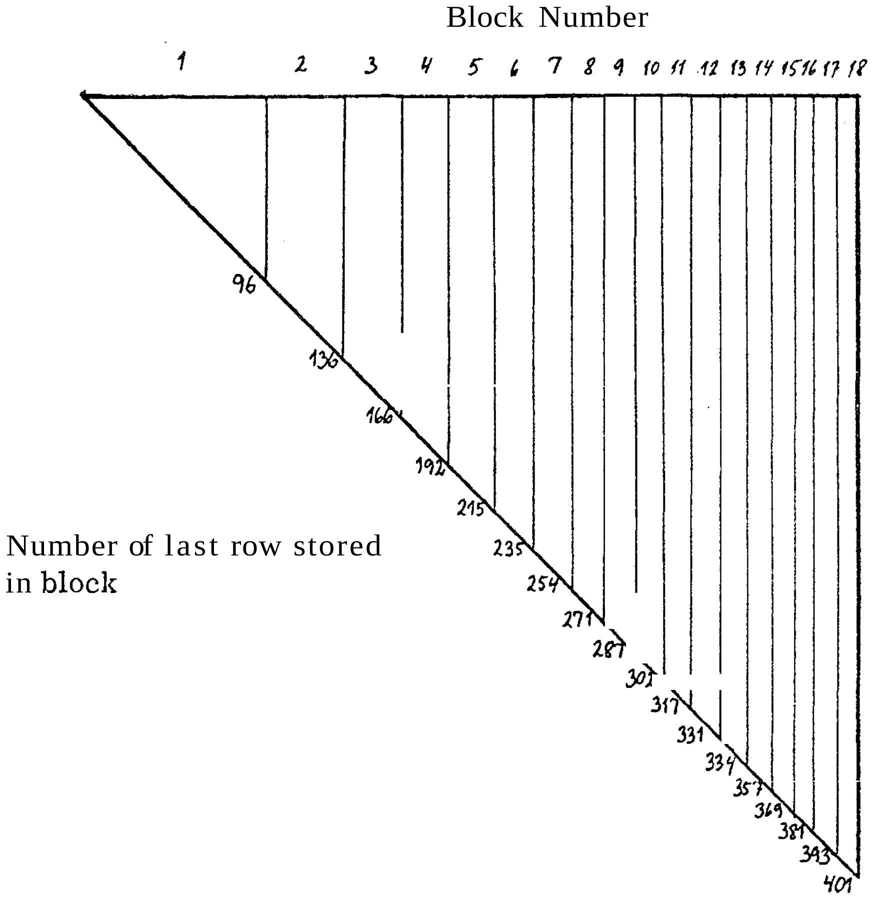


Figure 3. Blocking of 400 x 400 matrix

It is necessary, that two blocks can be stored simultaneously in the core storage, i.e. the maximum block size is then 225K or $(450/2) \cdot 1024/8 = 28800$ double precision coefficients. This number is then also the upper limit for the dimension of the normal equation matrix, N. (Another limit is set by the magnitude of the disk unit used. For the IBM model 3330 disk used here, N will have to be less than circa 5000).

We have in this program edition preferred to limit the total storage requirements to 252K (which for the present operative system gives a reasonable turn-around time). Thus, 90K can be used for the storing of the required two blocks and for the buffer area necessary for the transfer between core store and disk.

On a disk it is practical to block data in groups which occupy an integer number of tracks. We have then chosen to work with data partitioned into blocks of size 4800*8 bytes, covering three tracks and to use a buffer area of 1200*8 bytes. The total area occupied in core storage is hence 2*(38400) + 9600 bytes or nearly 86K. (The disk discussed is, as mentioned above, an IBM model 3330 disk). Figure 4 shows the number of tracks used as a function of the number of observations, N.

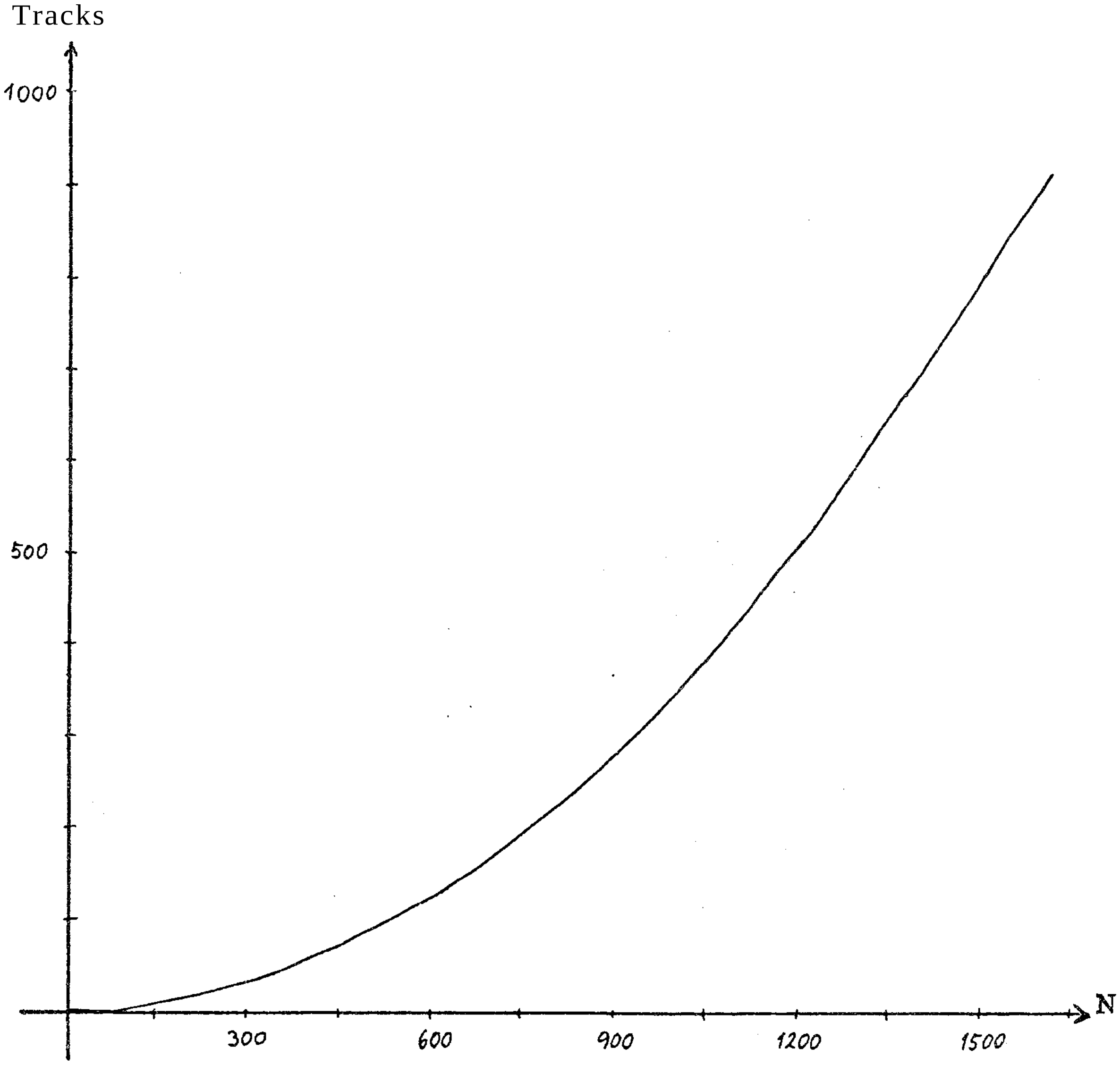


Figure 4. Number of tracks used on an IBM model 3330 disk unit for a varying number of unknowns, N (12800 bytes used on each track).

When a coefficient of the normal equation matrix $\overline{C}$ (eq. 12) is computed (subroutine PRED) it will first be stored in an array C of dimension 4700 (the last 100*8 bytes are used to hold two catalogues). Where the array C is filled up with as many columns as possible, the content will be transferred to the disk and stored in a direct access dataset (see IBM (1973, page 67)). The constant vector of observations, x is stored together with the coefficients, as if it was an extra column. The ficticious diagonal element of this column will contain the normalized square sum of the observations.

As mentioned above, the last 100*8 bytes of a block are used to hold two catalogues. The first catalogue contains the subscripts of the diagonal elements of the columns stored in the block. The second contains the subscript of the last zero element encountered in a column, starting the inspection of a column from the top. This catalogue may especially be used when $\overline{C}$ is a sparse matrix (e.g. when potential coefficients are not necessarily stored). In this program $\overline{C}$ will always be a full matrix, so the catalogue entries are just equal to zero. (Their value may be changed by the subroutine NES, in case singularities are encountered).

5.2 Solution of the Normal Equations and Computation of the Estimate of the Error of Prediction, Subroutine NES.

The equations are, as mentioned in Section 5.1, solved by first computing the upper triangular matrix L? (cf. (91)). This method is the well known Cholesky's factorization method.

We obtain, from (15) and (91) by a left multiplication with L^{-1}

$$(92)\quad L^{T}\cdot x = L^{-1}b = b'$$

The algorithm for the computation of the elements of L^{T}, ℓ_{ij} is

$$(93)\quad \ell_{ij} = \frac{1}{\ell_{ij}}\left(c_{ij} - \sum_{k=1}^{i-1} \ell_{ki}\,\ell_{kj}\right)$$

and nearly exactly the same for the computation of the elements b'_i of (92):

$$(94)\quad b'_i = \frac{1}{\ell_{ii}}\left(x_i - \sum_{k=1}^{i-1} \ell_{ik}\cdot b'_i\right),\ i=1,\ldots m$$

i.e. the algorithm (93) will compute (92), when x is regarded and stored as an extra column of the matrix $\overline{C}$. When b' has been computed, b can easily be obtained from (92) by a so called back-substitution procedure. We note, that we have (cf. eq. (6) and **(13)**):

$$(95) \quad C_s^T \overline{C}^{-1} C_s = C_s^T (L^T)^{-1} L^{-1} C_s = (L^{-1} \cdot C_s)(L^{-1} C_s).$$

Then, using the algo ithm **(93)** for j = m+1 with c , substituted for c_{ij}, we will obtain the quantity L–'C, for i = 1, ..., m. By defining an element $c_{m+1,m+1} = C_{ss}$ and using (93) for i = m+1 we will have computed the quantity $\tilde{m}_s^2$ (13).

The sub outine NES uses these algorithms for the computation of the vector b and the quantity $\tilde{m}_s^2$. The elements of L^T are stored in the positions on the disk, where earlier the coefficients of the upper triangular part of $\overline{C}$ were stored.

The matrix $\overline{C}$ is theoretically, always positive definite. Thus, mistakes may be made, which make $\overline{C}$ non positive definite. The Choleskys algorithm (93) will not work in this case, because the diagonal element of L', ℓ_{ii}, is computed by taking the square- root of equation (93), where both sides have been multiplied with ℓ_{ii}. The occurence of a negative quantity

$$\ell_{ii}^2 = c_{ii} - \sum_{k=1}^{i-1} \ell_{ki}^2$$

r

will not stop the execution of the program. NES will regard the column and corresponding row as deleted, and b_i will be put equal to zero.

Cholesky's method is very favorable numerically. But the proper use of the method requires that the sum of the products $\ell_{ki}\,\ell_{kj}$ in (93) are accumulated in a variable, which in this case would be in quadruple precision. The final product sum would then have to be rounded properly to double precision. Unfortunately rounding is not done by simply requiring the quadruple precision variable to be stored in a double precision variable, but supplementary statements have to be used. Thus, the solution vector b, is here obtained by computations performed in double precision only, which in this case anyway, gives a satisfactory number of significant digits.

The solution vector b, is obtained in $0.7 * \left(\frac{N}{100}\right)^3$ seconds, where N is the dimension of the normal equation matrix.

5.3 Transformation Between Reference Systems, Subroutine ITRAN.

We pointed out in Section 3, that it was necessary to transform the coordinates and measurements into a geocentric reference system. This transformation is performed for the coordinates, the deflections and the height anomaly by the subroutine ITRAN.

The subroutine uses the euclidian coordinates X, Y, and Z for a point with geodetic latitude φ and longitude λ and with the ellipsoidal height equal to zero.

These coordinates are then transformed into geocentric coordinates, X_1, Y_1, Z_1 by

$$(96)\quad \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \begin{Bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{Bmatrix} + (1+\Delta L)\cdot \begin{Bmatrix} 1 & \epsilon_1 & -\epsilon_2 \\ -\epsilon_1 & 1 & \epsilon_3 \\ \epsilon_2 & -\epsilon_3 & 1 \end{Bmatrix} \cdot \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix},$$

where (ΔX, AY, AZ) are the coordinates of the center of the old reference ellipsoid given in the new coordinate system, AL the scale change and (c,, c,, c,) the three infinitesimal rotations around the X, Y, Z axes respectively.

The new geodetic latitude φ_1 and longitude λ_1 of this point is computed using the ite ative procedure given in PG (p. 183). This computation will also furnish us with the change in the height anomaly, which is identical to the height of the point (X_1, Y_1, Z_1) above the new reference ellipsoid.

The change in the deflection components are then determined using the differences $\varphi_1 - \varphi$ and X, - X.

A contingent correction for systematical errors in the deflections or the height anomaly (cf. Section 3.2), specified by the changes $\delta\xi_0$, $\delta\eta_0$, $\delta\zeta_0$ in a point with coordinates φ_0, λ_0, is computed by the subroutine using the equations given in PG (eq. (5-59)).

5.4 Computation of the Normal Gravity, the Normal Potential and the Contributions from the Potential Coefficients. Subroutines GRAVC and IGPOT.

The normal gravity may be given in two ways. Either by a gravity formula or by specifying a normal gravity field, from which the gravity formulae then can be derived, (cf. PG, Chapter 2). The only gravity formulae which can be used is the international gravity formulae, PG (cq. (2-126) and (2-131)). The normal gravity fields which can bc used, are those for which the reference ellipsoid is an equipotential surface, i. e. it is specified by the values of kM, a, f and ω.

We may need to kuow the reference gravity in two situations. Firstly when free-air anomalies are computed using measured gravity values and secondly, when we want to compute the change in the gravity anomalies due to the use of a new reference system.

The subroutine GRAVC will compute and store the constants (PG, Section 2.10) necessary for the computation of the normal gravity in one or two reference systems. The constants used to compute the value of the normal potential (J_{2n}, $n \leq 5$ in eq. (26)) and the change in the latitude component of the deflection of the vertical ξ due to the curvature of the normal plumbline (PG(5-34)) are computed as well. When the height exceeds 25 km, the derivatives of the series (26) with respect to the latitude and the distance from the origin, will be used for the computation of the normal gravity and the change in ξ. Thus, this method of computation can, unfortunately, not be applied when the international gravity formula is used.

The values of the normal gravity, the normal potential and the change in ξ are computed by calling separate entries to the subroutine. The subroutine IGPOT computes the value of the potential W(P) and the three components of the gradient of the potential, the value of kM, a and ω, cf. eq. (25).

Let us, as usual, define V by

$$W = V + \frac{\omega^2}{2} (r \cdot \cos \theta)^2 .$$

The coefficient modification method is used for the computation of the values of V and the gradient of V. This method uses the fact, that the derivative of a harmonic function with respect to euclidian coordinates again is a harmonic function. The potential coefficients of the (three) new harmonic functions $D_X V$, $D_Y V$, and $D_Z V$ are computed by means of a recursion algorithm given in James (1969, eq. (3) and (4)). The recursion algorithm is identical to the algorithm used for the evaluation of the values of the solid spherical harmonics. This fact simplifies the computations very much. It furthermore makes it unnecessary to store the three sets of modified potential coefficients. They are computed for each call of the subroutine from the original potential coefficients. The algo ithm may easily be modified to compute higher order derivatives (without extra storage requirements). Thus, the algorithm may also be used in case the program is extended to use second order derivatives as observed quantities.

The value of the potential and the gradient is used to compute the residual observation $d_1 x_{21}$:

The value of the potential is used together with the value of the normal potential (as computed by GRAVC) to compute a residual height anomaly. The gradient is used to compute the residual gravity anomalies and deflection components:

$$(97a)\quad d_1\Delta g_i = \Delta g_i - \left(\left((D_X W)^2 + (D_Y W)^2 + (D_Z W)^2\right)^{\frac{1}{2}} - \gamma_i\right),$$

$$(97b)\quad d_1 \xi_j = \xi_j - \left(\arctan\left(D_Z W/\left((D_X W)^2 + (D_Y W)^2\right)^{\frac{1}{2}}\right) - \hat{\varphi}_j\right)$$

$$(97c)\quad d_1 \eta_k = \eta_k - \left(\arctan\left(D_Y W/D_X W\right) - \lambda_k\right) \cdot \cos\varphi_k \quad ,$$

where γ_i is the normal gravity, $\hat{\varphi}_j$ is the geodetic latitude plus a correction for the curvature of the plumbline and X, the longitude.

The components of the gradient used in (97a) are evaluated in a point with height equal to the orthometric height plus the distance h_0 between the reference ellipsoid and an equipotential surface of $U_1 = U + T_1$ with potential equal to the potential of the normal potential, U on the ellipsoid. The separation h_0 is computed by evaluating T_1/γ on the ellipsoid. The other gradients are evaluated in the height equal to the orthometric height.

5.5 Computation of,Euclidian Coordinates, Conversion of Angles to Radians, Subroutines EUCLID, RAD.

The subroutine EUCLID computes the euclidian (rectangular) coordinates for a point with geodetic coordinates p, X, h (ellipsoidal height) given in a reference system with semi-major axis a and second eccentricity e by the equations PG (5-3) and (5-5).

RAD converts angles given in either (1) degrees, minutes, arc seconds, (2) degrees and minutes, (3) degrees or (4) (400) grades into units of radians. Other options may easily be added.

5.6 Subroutines for Output Management and Prediction Statistics, HEAD, OUT and COMPA.

The output requirements are discussed in Section 7. The main requirement is, that a determination of $\tilde{T}$ must be as well documented as possible. ? may be computed in several ways, cf. Table 1. This implies, that the output may vary in just as many ways.

An array OBS is used for the storage of the observed quantities, the residual observations, the contributions from the different sets of observations and the predicted quantities. The storage sequence of these quantities is determined by BEAD, which also will print proper headings. The coordinates of an observed or predicted quantity and the quantities stored in OBS are printed on the line printer by OUT, which also will punch a part of this information when requested (see Section 7).

COMPA uses the content of OBS for the computation of prediction statistics. The difference between observed and predicted quantities are sampled in classes defined by a specified class width. The number of differences in each class is printed by COVA after the final predictions have been computed. The sampling is done separately for Δg, ξ and η. No sampling is done for ζ.

5.7 Subroutines for the Computation of Covariances, PRED and SUMK.

PRED computes:

(a) the vector $d_{i-1}C_s$ or $d_{i-1}C_i$, (cf. eq. (78a) and (82a)).
(b) a column of the upper triangular part of the normal equation matrix (eq. (80a)) d_iC_{ii} or
(cj the product sum $d_i s = d_iC_s^T \cdot b_i$, (cf. cq. (81a)).

The subroutine may theoretically work even when the observations (different from potential coefficients) are divided in more than two groups, as long as the total number of observations do not exceed 1600 minus the number of groups minus one.

When the observed quantity is a pair of deflections of the vertical it is very easy to compute the two corresponding columns of the upper triangular part of C at the same time. This is due to the similarity of the equations for the covariances (44)-(50)for ξ and η. This fact is used in the subroutine.

We mentioned in Section **3.1**, that it would facilitate the computations if the observations were grouped according to common characteristics, i.e., e.g. gravity anomalies on the surface of the Earth in the first group, deflections in the second group, gravity anomalies in 10 km's height in the third group, etc. The group characteristics (the type of observation and the quantity RP, ($7)) are stored in two arrays INDEX and P, which will also contain the subscripts of the first observation in the group within the total set of observations and a quantity related to the square root of the variance of the observations. This quantity is used for the scaling of the normal equations (in which all the diagonal elements will be equal to one).

The covariances are computed using the equations given in Section 2.2. Thus, for degree-variance model **3**, the covariances will be evaluated using the subroutine SUMK as well. This subroutine computes the sum of the infinite series

$$\sum_{\ell=3}^{\infty} \frac{1}{(\ell+B)} s^{\ell} P_{\ell}(t), \quad \sum_{\ell=3}^{\infty} \frac{1}{(\ell+B)} s^{\ell-1} D_t P_{\ell}(t) \text{ and } \sum_{\ell=3}^{\infty} \frac{1}{(\ell+B)} s^{\ell-2} D_t^2 P_{\ell}(t) ;$$

which are needed for the computation of the quantities $\breve{cov}_3(s_i, s_j)$, cf. eq. (39)and Tscherning and Rapp (1974, eq. (130)- (135)).

6. Input Specifications.

We can divide the input data in different (sometimes overlapping) groups:

(A) Data (generally true/false values of logical variables) determining the flow of the program (LTRAN, LPOT, LCREF, LGRID, LERNO, LSTOP),

(B) Data specifying selected input/output options,

(C) Data specifying the reference systems used for coordinates and observations,

(D) Data specifying the degree-variance model used,

(E) Data used for the determination of T (i.e potential coefficients, gravity values, deflections, etc.) and solutions to normal equations, and

(F) Data used to specify which quantities we want to predict.

The input flow is roughly sketched in Figure 5. The position of the integers 1-5 in the diagram indicates the beginning of the input data belonging to one of the 5 groups described below:

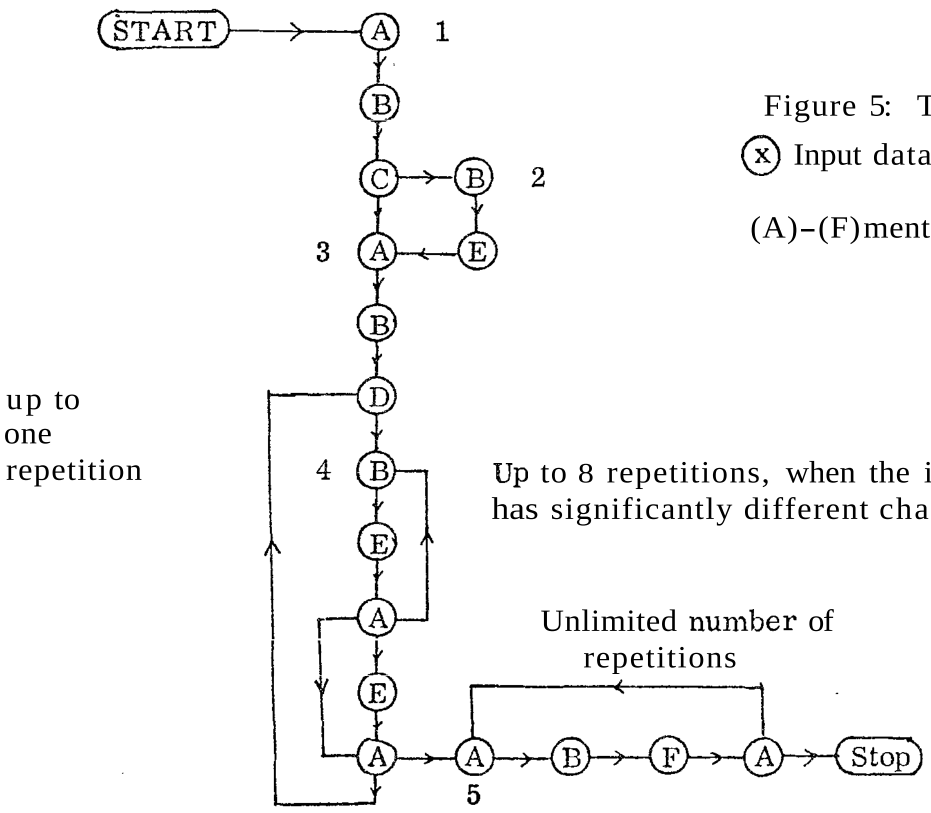


Figure 5: The Input Flow.
(x) Input data of category (A)-(F) mentioned in text.

The input consists exclusively of data on 80-column punch-cards. We will describe the content of each card, but not the format of the card. Instead, the format statement number will be given (in brackets) together with a two to five digit number, e.g. 3.013. This number will be used to identify the corresponding card shown in the input example, Appendix B.

We will divide the data in 5 categories:

(1) Data of type (A), (B) and (C), i.e. data describing the reference systems used,

(2) Data of type (A), (B) and (E), where the data of type (E) are the potential coefficients,

(3) Data of type (A), (B) and (D), i.e. data related to the degree-variance models,

(4) Data of type (A), (B) and (E), where the data of type (E) are observations of gravity anomalies, measured gravity values, height anomalies and deflections of the vertical,

(5) Data of type (A), (B) and (F).

The first digit in the identifying number will be the number of the category to which the card belongs. The other digits are used to indicate to which group or subgroup within the category the card belongs. In case an input situation depends on the content of e.g. the card 3.01 and there are two different input possibilities, the two cards will have the numbers 3.011 and 3.012 respectively. (Data of type (A) and (B) will, as mentioned above, in many cases be the true or false value of a logical variable. The function of a logical variable can be explained by writing e.g.: LE is a logical variable, which is true when XXX and false otherwise. Thus, we will in several cases below simply write LE = XXX.)

Category 1. (The numbers in brackets are, as mentioned above, the corresponding format statement numbers).

1.0 (105) The (true or false) values of five logical variables: LTRAN= the observations must be transformed to a new reference system, LPOT= potential coefficients are part of input data (observation data set x_1), LONEQ = output the coefficients of the normal equations on the line printer, LLEG = output a legend of the tables, which will be printed and LE = the standard deviations of the observations to be input (otherwise they are set equal to zero).

1.1 (103) A text of maximally 72 characters included in apostrophes, which identify the reference system of the observations.

1.2 (120) The semi-major axis (meters) and the inverse of the flattening of the Geodetic Reference System. The value of two logical variables, LPOTSD = the gravity are given in the Potsdam system and LGRS67 = the gravity data are given in the Geodetic Reference System, 1967.

In case the gravity data are not in the Potsdam system or in GRS 1967, input of:

1.21 (121) The product of the gravity constant and the mass of the Earth (kM) in units of $meters^3/sec^2$.

When LTRAN is true input of:

1.3 (131) The new semi-major axis (meters), the new kM ($meters^3/sec^2$), the inverse flattening, the translation vector (dX, dY, dZ) (meters), dL = one minus the scale factor, the three rotation angles (c,, c,, c,) (arc sec) and the value of a logical variable, LCHANG, which is true when the deflections and the height anomaly are to be corrected for a systematic error. (The correction must be given as a change $\delta\xi_0$, $\delta\eta_0$, $\delta\zeta_0$ at a point with coordinates p, X,, cf. Section 5.2).

When LCHANG is true, input of:

1.31 (133) φ_0 and λ_0 in degrees, minutes and arc seconds, $\delta\xi_0$, $\delta\eta_0$ in arc seconds and $\delta\zeta_0$ in meters.

Category 2. Data of this category are only input, when LPOT (card 1.0) is true. The values of kM and a, input on card 2.1, will have to be the best available estimates, cf. Section 3.2. They must be identical to the values input on card 1.3, when LTRAN is true.

2.0 (103) A text of maxiinally 72 characters, describing the source of the potential coefficients.

2.1 (137) kM ($meters^3$/sec"), a (meters), the normalized coefficient $\overline{C}_{2,0}$ multiplied by 10^6, the maximal degree of the coefficients and the value of a logical variable, LFM, which is false, when the coefficients $\overline{C}_{i,0}$ are punched on a separate card, and $\overline{C}_{ij}$, $\overline{S}_{ij}$ on the same card in a sequence increasing with i and j, and true when the coefficients are punched in the same sequence, on a number of cards, but with a fixed number of coefficients on each card. The first coefficient will, in both cases, have to be $C_{0,0}$ (even when this is zero) and all cards must have the same format, as given by 2.1. All the coefficients have to be fully normalized and multiplied by 10^6.

2.2 (103) The format of the cards on which the coefficients are punched (in brackets).

2.11 (format as given by 2.2). When **LFM** is false, the coefficients with $\overline{C}_{1,0}$ on one card and $\overline{C}_{1,1}$ and $\overline{S}_{1,1}$ on one card.

2.12 (format as given by 2.2). When **LFM** is true, the coefficients in a sequence increasing with i and j on a number of cards.

Category 3. We can select one of three anomaly degree-variance models, by giving the variable KTYPE the value 1, 2 or 3, cf. Section 2.2 eq. (30), (31) and (32).

3.0 (102) KTYPE

When KTYPE is equal to 3:

3.01 (107) IK = the variable B in equation (32).

The degree-variance model is then specified by giving

(a) the ratio R/R_e between the radius of the Bjerhammar sphere and radius of the mean Earth,

(b) the variance of the gravity anomalies on the surface of the Earth (VARDG2), (from which the constant A, in the equations (30)-(32) are computed, cf., e.g. equation (90)),

(c) either the "order" IMAX of the local covariance model to be used or a zero, which will indicate, that empirical anomaly degree-variances are used, and in this case

(d) the empirical anomaly degree-variances, given on the surface of the Earth, $\hat{\sigma}_a^E(\Delta g, \Delta g)$, (88).

3.1 (101) R/R_e, VARDG2 (in $mgal^2$) and IMAX.

When IMAX is equal to zero, input of the maximal degree, IMAXO of the degree-variances $\hat{\sigma}_a$ (Δg, Δg).

3.11 (102) IMAXO

3.12 (103) The format of the degree-variances. These must be punched on one or more cards, sequentially from degree 2 to IMAXO.

3.13 (format as given on card 3.12). The quantities $\hat{\sigma}_\ell^{\varepsilon}(\Delta g, \Delta g)$ in units of $mgal^2$.

Category 4. Input of up to 9 datasets with significantly different characteristics. One dataset can, for example, be two separate datasets punched differently, but both being gravity anomalies on the surface of the Earth. Another dataset may consist of mean gravity anomalies, all with the same format. Before each separate dataset, there will be input of 2 or more cards specifying the characteristics of the dataset.

All the records in a dataset must be punched in the same way. There are the following restrictions (or options): A station number may be punched. In this case it must be the first datafield on the card and maximally occupy seven positions. The next two datafields must contain the latitude and the longitude (in an arbitrary sequence). When the height is given, it must be punched in the next datafield.

The following (up to four) datafields will have to contain the observed quantity (or quantities in case of pairs of deflection components) and its standard deviation. When the observation is a pair of deflections, they have to be punched in the same sequence as the latitude and the longitude are punched. In the last datafield the value of the logical LSTOP has to be punched, generally false (= blank), but true for the last record in the dataset.

4.0 (103) The format of the records (in brackets).

4.1 (202) INO=1 when a station number is punched, 0 otherwise, ILA = the number of the datafield occupied by the latitude, ILO = the number of the datafield occupied by the longitude (ILA and ILO will be equal to 1, 2 or 3), an integer IANG specifying the units used for the latitude and the longitude (1 for degrees, minutes, arc. sec., 2 for degrees and minutes, 3 for degrees and 4 for 400-grades), IH= 0 when the height is zero and not punched and otherwise the number of the datafield in the record in which the height is punched (generally 3 or 4), IOBS1 = the datafield number of the first observation in the record, IOBS2 = the datafield number of the second observation (zero when there is only one observations), the value of an integer IKP, specifying the kind of observation: 1 for height anomalies, 2 for measured gravity or gravity anomalies (point or mean), 3 for pairs of deflections, 4 for the latitude component ξ and 5 for the longitude component η.

Then the ratio RP (87) between the sphere on which the observations are situated and the mean Earth radius and finally 5 logical variables: LPUNCH = punch observations together with the difference between the observed quantity and a possible contribution from the potential coefficients and a contribution from Collocation I, LWLONG = longitude is measured positive towards west, LMEAN = the gravity is a mean value, LSA = the standard deviations are the same for all observations, LKM = true when the height is in units of kilometers and false when the height is in meters.

When the observation is a height anomaly it will have to be given in units of meters, and when it is a deflection component in arc seconds. But when the observation is a gravity anomaly or a measured gravity quantity, it is possible to specify tivo constants DM and DA, which when DA is first added and the sum multiplied with DM will bring the observed quantity into units of mgal.

4.11 (203) DM, DA and a logical variable LMEGR, which is true, when the observation is a measured gravity value.

When LSA is true, the records of observations will not contain a standard deviation of the observed quantity, and the standard deviation will then have to be input separately:

4.12 (212) the standard deviation of the observations. Then the observations are input record after record, not exceeding a total number of 1598.

4.2 (format given on card 4.0). Input as specified on card 4.1, last record with LSTOP equal to true.

When LSTOP is true, a logical variable with the same name (i.e. LSTOP) is input. It is true when the last dataset is the final dataset used in Collocation I or II. The final card will hence have the value true (T) punched in the first datafield. On this card, in this case, the values of two other logical variables can be punched.

4.3 (230) The value of LSTOP, and when LSTOP is true, the values of two logical variables, LRESOL = input the solutions to the normal equations (they must then have been produced in a previous run of the program) and LWRSOL = punch the solutions to the normal equations.

When LSTOP is false, the input process will be repeated from card 4.0. When LSTOP is true and LRESOL is true, input of the solutions to the normal equations: First an identification card is read, then the solutions:

4.31 (361) Input of solutions, i.e. the cards produced in a previous run of the program, where the logical variable LWRSOL was true.

When the set of observations is the first one (the variable LC1 is false), input of a logical variable LCREF, else jump to 5.0.

4.4 (230) LCREF = a new set of observations, which will be used in collocation II, will have to be input,

For LCREF = true, jump back to 3.1.

Category 5. Data specifying quantities to be predicted. This specification will naturally have to be done in much the same way, as when the observations were specified. We need coordinates and some variables, which specifies tile type of quantity to be predicted. There are then two possibilities, which are distinguished by the true and false value of the variable LGRID.

When LGRID is false, we will proceed in exactly the same way as above, dealing with data of Category 4. The quantities to be predicted will be specified by a list of coordinates and 2 or more cards specifying format and type of quantity to be predicted.

The list of coordinates may in fact be a list of observed quantities, which we want to compare with the quantities to be predicted. If this is the case, a logical variable LCOMP has to be true.

When LGRID is true, the predictions will have to take place in points which form a grid. The south-west corner of the grid will have to be specified together with the distance between the mesh points in northern and eastern direction and the number of mesh points having the same longitude and the same latitude.

5.0 (200) Input of the logical values of LGRID, LERNO = estimate of error of prediction is wanted and LCOMP = compare predicted and observed quantities.

First time LCOMP is true, two constants used to specify the sampling width for a frequency distribution of the difference between observed and predicted gravity anomalies (VG) and deflections (VF) must be input (e.g. equal to 2.0 mgal and 0.5 arc. sec.). (The differences are sampled in 21 groups.)

5.01 (203) VG and VF.

When LGRID is true:

5.02 (201) Coordinates (latitude and longitude in degrees and minutes) of the south-west corner of the grid, the increments in latitude and in longitude (minutes), the number of increments in northern and in eastern direction, the value of the IKP giving the type of quantity to be predicted (see 4.1), RP (see 4.1), LMAP= print the predicted quantities on the line printer with all values which are predicted in points with the same latitude on one line and all values predicted in points with the same longitude above each other, LPUNCH= punch coordinates, predicted quantity and when LERNO is true the estimated error, LMEAN= gravity to be predicted is a mean value.

When LGRID is true, jump to card 5.1.

Now, when LGRID is false, we may input lists of coordinates just as above:

5.030 as 4.0 (format of records)

5.031 as 4.1, with the following changes; When LCOMP is true, LPUNCH will mean the same as in 4.1 and the error of prediction will be punched when LERNO is true. When LCOMP is false, the predicted quantity as given in the new and in the old reference system will be punched together with the error of prediction, when LERNO is true. The logical variable LSA has no function in this phase of the computations.

When no observed quantity is contained in the record, both IOBS1 and IOBS2 will have to be put equal to zero. Thus, in this case the record will have to contain the height. (The program requires the presence of at least one datafield between the datafields occupied by the latitude and the longitude and the datafield occupied by the logical variable LSTOP).

When IKP = 2 (we are predicting gravity quantities):

5.032 as 4.11

5.033 input as specified by 5.030.

5.1 The value of LSTOP, true when no more quantities are to be predicted.

When LSTOP is false, jump to 5.0.

An input example is printed in Appendix B.

7. Output and Output Options.

The output from the FORTRAN program has been designed with the purpose, that the determination of $\tilde{T}$ and subsequent predictions should be as well documented as possible. This means, that nearly everything, which is used as input also will be output.

There are a few exceptions:

(a) data of type (A) and (B) (cf. Section 6) are not printed,

(b) the potential coefficients are not printed,

(c) a measured gravity value is not printed, but the corresponding free-air anomaly is,

(d) more than two decimal digits of coordinates given in minutes or seconds and of observations are generally not reproduced.

With these exceptions all input of type (C) to (F) are printed with proper headings on the line printer.

We will now distinguish between non-optional and optional output. The output can be made on two units, unit 6 the line printer and unit 7 the card punch. Non-optional output is output on the line printer exclusively.

Non-optional output:

- A program identification is printed giving date of program version.
- The used mean Earth radius and the reference gravity used on the sphere in equations (20)-(22) is printed.
- The equatorial gravity and the potential of the reference ellipsoid as computed from the constants specifying the reference systems.
- The residual observations $d_{i-1}x_i$, and if meaningful: the contribution from the datum transformation, from the potential coefficients, the first dataset (Collocation I) and the second dataset (Collocation II) and the sum of these contributions,
- The predicted quantity and if meaningful: the contributions from the datum transformation, the potential coefficients, Collocation I and II.
- The solutions to the normal equations,
- The estimated variance of the residual observed and predicted quantities, and
- Error messages in case e.g. certain array limits are exceeded.

Optional output on the line-printer:

- A legend of the labels of observations and predictions, (LLEG= true),
- The difference between observed and predicted quantities (LCOMP= true),
- The estimated error of prediction (LERNO= true),
- A primitive "map" of the predictions (LGRID= true and LMAP= true). (The predicted quantities multiplied by 100 will be printed with the values predicted in points with the same latitude on one line and the values predicted in points with the same longitude above each other, see the "map", Appendix C, page 125.)
- Mean value and variance of difference between observed and predicted quantities and table of distribution of the differences samples according to specified sample width (LCOMP is true).

Optional output on the card punch:

- The solutions to the normal equations b_1 and b_2 (LWRSOL= true)
- the observed quantities and the residual observations (LPUNCH= true),
- the predicted quantities, the estimated error (LERNO= true), the difference between observed and predicted quantities (LCOMP= true), and when LCOMP is false, the predicted quantity in the original and the new reference system.

The solutions to the normal equation can be used as input to the program, cf. Section 6, input specification No. 4.31.

An example of the output on the line printer is given in Appendix C.

8. Recommendations and Conclusions.

A development of a computer program as the one presented here is a task, which can be continued for years. But at some point it is necessary to stop and present a fully documented program version, even if it is obvious that improvements can be made.

Most of the recent ideas and investigations in the field of least squares collocation are used in the program. Hence, the program may principally be used for

- the determination of an approximation to the anomalous potential, $\tilde{T}$ and

- prediction and filtering of gravity anomalies, deflections and height anomalies.

The determination of $\tilde{T}$ may be improved in several ways. The program should be changed so that other types of data as e. g. density anomalies, satellite orbit perturbations, and gravity gradients can be used as observations and predictions. The program should also be able to predict potential coefficients. The covariance models, which can be used in the program are all isotropic. The use of a non-isotropic covariance model may improve the determination of $\tilde{T}$.

In Section 3.2 it was pointed out, that the data had to be given in a geocentric refe ence system. Thus, the necessary translation parameters may be estimated by including these quantities as parameters X, cf. eq. (7) and (9), Tscherning (1973) and Moritz (1972, Section 6).

It is also possible to add new data to an original set of observations, without having to compute and invert the full covariance matrix. This type of computation is denoted sequential collocation cf. Moritz (1973). This feature may very easily be incorporated in the program, especially because of the flexible design of the subroutine NES (cf. Section 5.2 and the comments given to the subroutine in Appendix A).

The determination of potential coefficients and datum shift parameters may also be incorporated without difficulties. But the other proposed improvements can not be made before the theoretical background and the necessary algorithms have been developed.

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Appendix

A. The FORTRAN IV program.

B. An input example.

C. An output example.

Appendix A.

The FORTRAN IV program.

The program is written in the language FORTRAN IV, cf. IBM (1973). It may be run on an IBM model. 370 computer equipped with an IBM model 3330 disk unit. The program may be compiled and executed using the catalogued procedure FORTXCLG, cf. IBM (1972, p. 89) using the following job control language statements:

```
//     EXEC FORTXCLG,PARM.FORT='OPT(2)',
//   TIME.FORT=(,30),REGION=252K
//FORT. SYSIN DD *
           .
           .
    program statements
           .
           .
//GO.  FT08F001 DD DSN=DASET,UNIT=SYSDA,
//       SPACE=(12800,920),DISP=(,DELETE),DCB=(DSORG=DA,BUFNO=1)
//GO.  SYSIN DD *
           .
           .
    input data
/*
//
```

```
C
C PROGRAM GEODETIC COLLOCATION, VERSION 20 APR, 1974, FORTRAN IV, (IBM
C 360/70). PROGRAMMED BY C.C.TSCHERNING, DANISH GEODETIC INSTITUTE/ DEP.
Q GEODETIC SCIENCE, OSU.
C THE PROGRAM COMPUTES AN APPROXIMATION TO THE ANOMALOUS POTENTIAL OF
C THE EARTH USING STEPWISE LEAST SQUARES COLLOCATION. THE METHOD REQUI-
C RES THE SPECIFICATION OF (1) ONE Oh TWO (AND IN A SPECIAL CASE THREE)
C SETS OF OBSERVED QUANTITIES WITH KNOWN STANDARD DEVIATIONS AQD (2) ONE
C OR TWO COVARIANCE FUNCTIONS.
C THE COVARIANCE FUNCTIONS USED ARE ISOTROPIC. THEY ARE SPECIFIED BY A
C SET OF EMPIRICAL ANOMALY DEGREE-VARIANCES OF PEGREE L E S THAN AN
C INTEGER VARIABLE IMAX, AND EY A ANOMALY DEGREE-VARIANCE MODEL FOR THE
C DEGREE-VARIANCES OF DEGREE GREATHER THAN IMAX.
C THE OBSERVATIONS MAY BE POTENTIAL COEFFICIENTS, MEAN OR POINT GRAVITY
C ANOMALIES, HEIGHT ANOMALIES AND DEFLECTIONS OF THE VERTICAL. A FIL-
C TERING OF THE OBSERVATIONS TAKES PLACE SIMULTANEOUSLY WITH THE DETER-
C MINATION OF THE ANOMALOUS POTENTIAL.
C THE DETERMINATION IS MADE IN A NUMBER OF STEPS EQUAL TO THE NUMBER OF
C SETS OF OBSERVATIONS. WHEN POTENTIAL COEFFICIENTS AFE USED, WILL THE-
C ESE FORM A SEPERATE SET AND THE TOTAL NUMBER OF SETS MAY IN THIS CASE
C AMOUNT TO THREE.
C EACH DATASET (EACH STEP) WILL DETERMIUE A HARMONIC FUNCTION, AND THE
C ANOMALOUS POTENTIAL WILL BE EQUAL TO THE SUM OF THEESE (MAXIMALLY
C THREE) FUNCTIONS.
C POTENTIAL COEFFICIENTS WILL DETERMINE A FUNCTION EQUAL TO THE COEFFI-
C CIENTS MULTIPLIED BY THE CORRESPONDING SOLID SPHERICAL HARMONICS. THE
C UP TO TWO SETS OF DATA DIFFERENT FRPM POTENTIAL COEFFICIENTS WILL
C EACH RE USED TO DETERMINE CONSTANTS B(I). THE CORRESPONDING HARMONIC
C FUNCTIONS ARE THEN EQUAL TO THEESE CONSTANTS MULTIPLIED BY TYE COVA-
C RIANCE BETWEEN THE OBSERVATIONS AND THE VALUE OF THE ANOMALOUS POT-
C ENTIAL IN A POINT, P.
C THE MAIN FUNCTION OF THE PROGRAM IS, BESIDES THE COMPUTATION OF THE
C CONSTANTS B(I S,THE PREDICTION OF THE QUANTITIES ZETA, DELTA G, KSI
C AND ETA IN POINTS Q. THE PREDICTED VALUE IS EQUAL TO THE PRODUCT SUM
C OF B(I) AND TEE COVARIANCE EETWEEN OBSERVATION NO.I AND THE QUANTITY
C TO BE PREDICTED.
C
C  REF(A): TSCHERNING,C.C. AND R.H.RAPP: CLOSED COVARIANCE EXPRESSIONS
C          FOR GRAVITY ANOMALIES, GEOID UNDULATIONS, AND DEFLECTIONS OF
C          THE VERTICAL IMPLIED BY ANOMALY DEGREE-VARIANCE MODELS. DEP-
C          ARTMENT OF GEODETIC SCIENCE, THE OHIO STATE UNIVERSITY,
C          REPORT NO. 208, 1974.
C  REF(B): TSCHERNING,C.C.: A FORTRAN IV PROGRAM FOR THE DETERMINATION
C          OF THE ANOMALOUS POTENTIAL USING STEPWISE LEAST SQUARES COL-
C          LOCATION, DEPARTMENT OF GEODETIC SCIENCE, THE OHIO STATE UNI-
C          VERSITY, REPORT NO. 212, 1974.
C  REF(C): HEISKANEN W.A. AND H.MORITZ: PHYSICAL GEODESY, 1967.
C  REF(D): JAMES, R.W.: GEOPHYS.J.R.ASTR.SOC.,(1969) 17, 305.
C
      IMPLICIT INTEGER(I,J,K,M,N), LOGICAL(L),REAL *8(A-H,O-Z)
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C
      COMMON/PR/SIGMA(250),SIGMAO(250),B(1600),P(42),
     *SINLAT(1600),COSLAT(1600),RLAT(1600),RLONG(1600),COSLAP,SINLAP,
     *RLATP,RLONGP,RP,PRETAP,PREDP,PW,LONECO,LNKSIP,LNETAP,LDEFVP,LNDFP,
     *LGRP,LNGR,LKEQ1,LKEQ3,LKNE1,IV1,NI,NR,KTYPE,INDEX(42)
C IN /PR/ AKE STORED: DEGREE-VARIANCES (SIGMA,SIGMAO), THE CONSTANTS
C B(I), TWO CATALOGES OF OBSERVATIONS (P AND I NDEX), LATITUDE AND COS,
C SIN HEREOF AND LONGITUDE OF THE OBSERVATION POINT (SINLAT,COSLAT,PLAT,
C RLONG), CORRESPONDING QUANTITIES FOR POINT OF PREDICTION (P), RATIO RP
C BETWEEN RADIUS OF SPHERE ON WHICH P IS SITUATED PND RB, TWO VARIABLES
C IN WHICH PREDICTIONS ARE ACCUMULATED (PREDP,PRETAP), A QUANTITY RELA-
C TED TO THE VARIANCE OF THE OPSERVATIONS OR PREDICTIONS (PW), LOGICAL
C VARIABLES USED TO DISTINGUISH BETWEEN DIFFERENT PREDICTION SITUATIONS
C AND COVARIANCE MODELS.
      COMMON /CRW/WOBS(1600)
C IN /CPW/ ARE STORED THE APRIORI STANDARD DEVIATIONS AS LONG AS THEY
C APE NEEDED. THE STORAGE LOCATIONS ARE LATER USED FOR OTHER PURPOSES.
      COMMON /EUCL/X,Y,Z,XY,XY2,DISTO,DIST2
C IN //EUCLID/ APE STORED: THE EUCLIDIAN COORDINATES OF A POINT, THE
C DISTANCE AND THE SQUARE OF THE DISTANCE FROM THE Z- AXIS XY, XY3 AND
C THE DISTANCE AND THE SQUARE OF THE DISTANCE FROM THE ORIGIN DISTO AND
C DIST2.
      COMMON /NESOL/C(4700),NCAT(100),ISZE(100),NBL(310),MAXBL,IQ
C IN /NESOL/ ARE STORED: THE ARRAY C USED TO TRANSFER THE COEFFICIENTS
C OF THE NORMAL EQUATIONS AND THE SOLUTIONS TO AND FROM DISK-STORAGE,
C NCAT, ISZE AND NBL HOLDS INFORMATION ABOUT THE STORAGE SEQUENCE OF THE
C COLUMNS, MAXBL IS THE NUMBER OF BLOCKS OF SIZE C+NCAT+ISZE USED ON THE
C DISK* IQ POINTS ON THE TRACK ON THE DISK AREA IN WHICH DATA IS TO EE
C STOPED OR RETRIVED.
      COMMON/OUTC/K2,K3,K4,IU,K21,IU1,IANG,LPUNCH,LOUTC,LNTRAN,LNERNO
     *,LK30
      COMMON /CHEAD/IA,IB,IH,IP,IT,IA1,IB1,IP1,IT1,IC1,IC11,K1,IOBS1,
     *IOBS2,LPOT,LC1,LC2,LCREF,LKM
C IN /OUTC/ AND /CHEAD/ ARE STORED INFORMATION USED TO HAYDLE THE DIF-
C FERENT I/O SITUATIONS.
      COMMON/OBSER/OBS(20)
      COMMON /DCON/DO,D1,D2,D3
      COMMON /SCK/IK,IKO,IK1,IK2,IKA
C
      DIMENSION IMAP(400),FMT(9),WP(5),C1(1600),C2(1600),C3(1500)
     *,COFF(630)
      EQUIVALENCE (C(1),C1(1)),(C(1601),C2(1)),(C(3201),C3(1))
C
      DATA RE,GM/6371.0D3,3.98D19/,LNEQ,LT
     *,LDEFF,LF,LGRID,LFRNO,LCOMP,LCOM,LWLONG,LPRED/2*.TRUE.,8*.FALSE.
     */,NO,NAI,NLA,IC,IS,ISO,II,JR/6*0,2*2/
C
C THE DIRECT-ACESS FILE DEFINED HERE IS USED FOR THE STORAGE OF THE
C COEFFICIENTS OF THE NORMAL EQUATIONS. IT CAN HAVE UP TO 310 RECORDS
C OF NT*3200 4-BYTE WORDS EACH. THE LIMIT IS ONLY DETERMINED BY THE DI-
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C MENSION OF THE ARRAY NBL (IN THE COMMON BLOCK /NESOL/).
      DEFINE FILE 8(920,3200,U,IQ)
C
C INITIALIZATION OF VARIABLES, WHICH ARE IN COMMON BLOCKS.
      DO = 0.0D0
      D1 = 1.0D0
      D2 = 2.0D0
      D3 = 3.0D0
      P(1) = DO
      P(21) = DO
      COSLAT(1600) = D1
      SINLAT(1600) = DO
      RLAT(1600) = DO
      RLONG(1600) = DO
      CLA = PO
      WP(1) = RE**3/GM
      W = RE**2/GM*206264.806D0
      WP(2) = D1
      WP(3) = W
      WP(4) = W
      WP(5) = W
      BT = O
      IP = O
      LNERNO = LT
      LCPEF = LF
      LC1 = LF
      LC2 = LF
      INDEX(1) = 0
      DO 1200 I = 1, 250
 1700 SIGMAO(I) = D0
C
C HEADINGS AND DEFINING CONSTANTS.
      WRITE(6,104)
  104 FORMAT('1GEODETIC COLLOCATION,VERSION 20 APR 1974.'//)
      WRITE(6,113)
  113 FORMAT('ONOTE THAT THE FUNCTIONALS ARE IN SPHERICAL APPROXIMATION'
     */' MEAN RADIUS = RE = 6371 KM AND MEAN GRAVITY 981 KGAL USED.')
C
C INPUT OF    5 LOGICAL VARIABLES, LTRAN = COORDINATES ARE TO SE TRANS-
C FORMED TO NEW RFFERENCE SYSTEM, LPOT = POTEYTIAL COEFFICIENTS ARE TO
C BE USED AS FIRST SET OF OBSERVATIONS, LONEQ = OUTPUT COEFFICIENTS OF
C NORMAL EQUATIONS ON UNIT 6, LLEGN = OUTPUT LEGEND OF TABLES OF OBSER-
C VATIOYS OR PREDICTIONS AND LE = TAKE ERRORS OF OBSERVATIONS INTO AC-
C COUNT.
      READ(5,105)LTRAN,LPOT,LONEQ,LLEG,LE
  B05 FORMAT(5L2)
      LNTRAN = .NOT.LTRAN
      LNPOT = .NOT.LPOT
      IF (.NOT.LE) WRITE(6,118)
  118 FORMAT(' ERRORS IN OBSERVATIONS ARE NOT TAKEN INTO ACCOUNT.')
```

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C
      IF (LLEG) WRITE(6,114)
  114 FORMAT('OLEGEND OF TABLES OF OBSERVATIONS AND PREDICTIONS:',/,
     *' OBS = OBSERVED VALUE (WHEN BOTH COMPONENTS OF DEFLECTIONS ARE'/,
     *' OBSERVEC ETA BELOW KSI), DIF =THE DIFFERENCE BETWEEN OBSERVED'/,
     *' AND PREDICTED VALUE, WHEN PREDICTIONS ARE COMPUTED AND ELSE'/,
     *' THE RESIDUAL OBSERVATION, ERR = ESTIMATED ERROR OF PREDICTION'/,
     *' TRA = CONTRIBUTION FROM DATUM TRANSFORMATION, POT = CONTRI-'/,
     *' BUTION FROM POTENTIAL COEFFICIENTS, COLL = CONTRIBUTION FROM'/,
     *' COLLOCATION DETERMINED PART OF ESTIMATE, WHEN THERE ONLY HAS'/,
     *' PEFY USED ONE SET OF OBSERVATIONS (DIFFERENT FROM POT.COEFF.)'/,
     *' COLL1 = CONTRIBUTION FROM ESTIMATE OF ANOMALOUS POT. DETER-'/,
     *' MINED FSOM FIRST SET OF OBSERVATIONS, COLL2 = CONTRIBUTION'/,
     *' FROM ESTIMATE OBTAINED FROM SECOND SET OF OBSERVATIONS, PRED='/,
     *' PREDICTED VALUE IN NEW REFERENCE SYSTEM, WHEN PREDICTIONS ARE'/,
     *' COMPUTED AND ELSE THE SUM OF THE CONTRIBUTIONS FROM THE POT.'/,
     *' COEFFICIENTS AND FIRST ESTIMATE OF ANOMALOUS POTENTIAL. AND'/,
     *' PRED-TRA = PREDICTIPN PR SUM OF CONTRIBUTIONS IN THE OLD RE-'/,
     *' FERENCE SYSTEM.')
C
C INPUT OF DATA OF REFERFYCF SYSTEM.
      WRITE(6,106)
  106 FORMAT('OREFERENCE SYSTEM:')
C   INPUT OF TEXT DESCRIBING REFERENCE SYSTEM (MAX.72 CHARACTERS).
      READ(5,103)FMT
      WRITE(6,FMT)
C INPUT OF SEMI-MAJOR AXIS (METERS), 1/FLATTENING, VALUE OF TWO LOGICAL
C VARIABLES, LPOTSD = GRAVITY IN POTSDAM SYSTEM AND LGRS67 = GPAVITY RE-
C FER TO GRS 1967.
      READ(5,120)AX1,FO,LPOTSD,LGRS67
  120 FORMAT(F10.1,F10.5,2L2)
      F1 = D1/FO
      E21 = F1*(D2-F1)
      IF (LPOTSD.OR.LGRS67) GO TO 1021
C INPUT OF GM OF PEFERENCF-SYSTFM OF OBSERVATIONS.
      READ(5,121)GM1
  121 FORMAT(D15.8)
 1021 IF(.NOT.LGRS67) CALL GRAVC(AX1,F1,GM1,0,LPOTSD,UREFO,GREF)
      IF (LGRS67) CALL GRAVC(6378160.0D0,D1/298.2471700,3.98603D14,0,
     *LF,UREFO,GREF)
      WRITE(6,122)AX1,FO,GREF,UREFO
  122 FORMAT('OA    =',F10.1,' M'/,
     *' 1/F =',F10.5/,
     *' REF.GRAVITY AT EQUATOR =',F12.2,' MGAL'/,
     *' POTENTIAL AT REF.ELL.  =',F12.2,' M**2/SEC**2'/,
     *' GRAVITY FORMULA:')
      IF (LPOTSD) WRITE(6,123)
      IF (.NOT.(LPOTSD.OR.LGRS67))WRITE(6,124)GM1
  123 FORMAT(' INTERNATIONAL GRAVITY FORMULA, POTSDAM SYSTEM.')
  124 FORMAT(' COEFFICIENTS COMPUTED, USING GM =',D15.8/)
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      IF (LGRS67)WRITE(6,125)
  125 FORMAT(' GRS 1967 USED.')
C
      LNTP = LNPOT.AND.LNTRAN
      IF (LNTP) GO TO 1030
      IF (LNTRAN) GO TO 1097
C
C INPUT OF TRANSFORMATION ELEMENTS: NEW SEMI-MAJOR AXIS (AX2, METERS),
C NEW GM (GM2, METERS**3/SEC**2), 1/FLATTENING, THE COORDINATES OF THE
C CENTER OF THE REFERENCE ELLIPSOID ( THE TRANSLATION VECTOR) (DX,DY,DZ)
C IN METERS, THE CHANGE DL IN SCALE, AND THE ROTATION ANGLES EPS1, EPS2,
C EPS3 AROUND THE X,Y,Z AXES IN ARCSEC. THEN THE VALUE OF A LOGICAL
C VARIABLE LCHANG, WHICH IS TRUE, WHEN WHEN THE DEFLECTIONS AND THE
C HEIGHT ANOMALIES (BUT NOT THE COORDINATES) HAVE TO BE CHANGED. THIS
C CHANGE MUST BE GIVEN AS A CHANGE IN THE DEFLECTIONS AND THE HEIGHT
C ANOMALY IN A POINT WITH COORDINATES (LATO, LONGO).
C THE COORDINATES MUST BE INPUT IN DEGREES, MINUTES AND SECONDS, FOL-
C LOWED BY THE TRANSFORMATION ELEMENTS IN KSI, ETA AND ZETA (DKSIO,
C DETAO,DZETAO) IN ARCSEC AND METERS.
C
      READ(5,131)AX2,GM2,F2,DL,DX,DY,DZ,EPS1,EPS2,EPS3,LCHANG
  131 FORMAT(F10.1,D15.7,F10.5,D10.2/3F7.1,3F6.2,L2)
      WRITE(6,132)AX2,GM2,F2,DL,DX,DY,DZ,EPS1,EPS2,EPS3
  132 FORMAT('0 NEW A        NEW GM            NEW 1/F'/,
     *F10.1,D15.7,F10.5,//
     *'     DL          DX         DY         DZ   ',/,D10.2,3F7.1,//,
     *'   EPS1   EPS2   EPS3',/,3F6.2)
      F2 = D1/F2
      E22 = F2*(D2-F2)
      CALL GRAVC(AX2,F2,GM2,15,LF,UREFO,GREF)
      WRITE(6,135)GREF,UREFO
  135 FORMAT('0 NEW REF. GRAVITY AT EQUATOR=',F12.2,' MGAL',/
     *'   NEW POTENTIAL AT ELLIPSOID =',F12.2,' M**2/SEC**2',/)
      IF (.NOT.LCHANG) GO TO 1022
C
      READ(5,133)IDLAT,MLAT,SLAT,IDLON,MLON,SLON,DKSIO,DETAO,DZETAO
  133 FORMAT(2I3,F6.2,2I3,F6.2,3F6.2)
      WRITE(6,134)IDLAT,MLAT,SLAT,IDLON,MLON,SLON,DKSIO,DETAO,DZETAO
  134 FORMAT('0DEFLECTIONS AND HEIGHT ANOMALIES CHANGED IN',/,
     *'   LATITUDE       LONGITUDE BY DKSI   DETA   DZETA',/,
     *2I3,F6.2,2I3,F6.2,3F7.2)
      CALL RAD(IDLAT,MLAT,SLAT,RLATO,1)
      CALL RAD(IDLON,MLON,SLON,RLONGO,1)
 1022 CALL ITRAN(DX,DY,DZ,EPS1,EPS2,EPS3,DL,AX2,E22,RLATO,RLONGO,
     *DKSIO,DETAO,DZETAO,LCHANG)
      GO TO 1008
C
 1097 E22 = E21
      AX2 = AX1
 1008 IF (LNPOT) GO TO 1030
```

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C
C INPUT OF TFXT DESCRIBING SOURCE OF THE POTENTIAL COEFFICIENTS (MAX. 73
C CHARACTERS).
      READ(5,103)FMT
      WRITE(6,130)
  130 FORMAT('OSOURCE OF THF POTFNTIAL COEFFICIENTS USED:')
      WRITE(6,FMT)
C
C INPUT OF GM (METERS**3/SEC**2), A (METERS), THE NORMALIZED COEFFICIENT
C OF DEGREE TWO AND OPCEP ZERO (THE SECOND DEGREE ZONAL HARMONIC) MUL-
C TIPLIED BY 1.0D6, THE MAXIMAL DEGREE OF THE COEFFICIENTS, A LOGICAL
C VARIABLE, TPUE WHEN THE COEFFICIEYTS ARE PUNCHED WITH A FIXED NUMBER
C ON EACH CARD AND FALSE, WHEN THF COEFFICIENTS OF THE ZONAL HARMONICS
C ARE PUNCHED SEPERATLY ON ONE CARP AND TYE OTHER COEFFICIENTS WITH THE
C COEFFICIENTS OF THF SAMF ORDER AND DEGREE ONE ONE CARD. IN BOTH CASES
C MUST THE COEFFICIENTS EF PUNCHED ACCORDING TO INCREASING DEGREE AND
C ORDER. ALL COEFFICIEYTS MUST EE NORMALIZED AND MULTIPLIED BY 1.0D6.
      READ(5,137)GMP,AX,COFF(5),NMAX,LFM
  137 FORMAT(D15.8,F11.1,F10.4,I4,L2)
      WRITE(6,138)GMP,AX,COFF(5),NMAX
  138 FORMAT('0    GM                A          CC'FF(5)   MAX.DEGREE',/
     *D15.8,F11.1,F10.4,I4)
      IF (NMAX.LT.24) GO TO 1009
      WRITE(6,140)
  140 FORMAT(' YMAX TOO BIG.')
      GO TO 9999
C
 1009 NZ = (NMAX+1)**2
C INPUT OF FORMAT OF COEFF.
      READ(5,103)FMT
      IF (LFM) CO TO 1225
      READ(5,FMT)(COFF(I), I = 6, 9)
      JM = 9
      DO 1224 J = 3, NMAX
      JN = JM+1
      JM = JN+2*J
      READ(5,FMT)COFF(JN)
      JN = JN+1
 1224 READ(5,FMT)(COFF(I), I = JN, JM)
      GO TO 1226
 1225 READ(5,FMT)(COFF(I), I = 6, N2)
 1226 DO 1034 1 = 1, 4
      COFF(I+N2) = DO
 1034 COFF(1) = GO
C
      CALL IGPOT(GMP,AX,COFF,N2+4,NMAX)
      IF (.NOT.LTRAN) CALL GRAVC(AX,F1,GMP,15,LF,UREFO,GRFF)
C
C COLLOCATION SECTION: INITIALIZATION OF VARIABLES.
C
```

```
 1050 N = 0
      WRITE(6,109)
  109 FORMAT('0START OF COLLOCATION I:')
C INPUT OF THE INTEGEP KTYPE DETERMINING TYPE OF DEGREE- VARIANCE MODEL
C USED FOR DEGREE-VARIANCES OF DEGREE GREATHER THAY IMAX (SEE BELOW).
C KTYPE MAY BE EQUAL TO 1, 2 AND 3 CORRESPONDING TO THE DEGREE-VARIANCE
C MODELS 1, ? AND 3, CF. REF(B), SECTION 2.2.
      READ(5,102)KTYPE
      LKEQ1 = KTYPE.EQ.1
      LKEQ3 = KTYPE.EQ.3
      LKNE1 = .NOT.LKEQ1
      IF (KTYPE.LT.3) GO TO 1036
      READ(5,107)IK
  107 FORMAT(I4)
      IF (KTYPE.LE.0 .OR. KTYPE.GE.4) GO TO 9999
C INITIALIZATION OF VARIABLES IN COMMON BLOCK /SCK/.
      IK0 = IK-1
      IK1 = IK+1
      IK2 = IK+2
      TKA = IK2*IK1
C
 1036 WRITE(6,141)
  141 FORMAT('0THE MODEL ANOMALY DEGREE-VARIANCES ARE EQUAL TO'/,
     *' A*(I-1)/')
      GO TO (1037,1038,1039),KTYPE
 1037 WRITE(6,142)
  142 FORMAT('+',9X,'I.')
      GO TO 1000
 1038 WRITE(6,143)
  143 FORMAT('+',9X,'(I-2).')
      GO TO 1000
 1039 WRITE(6,144)IK
  144 FORMAT('+',9X,'((I-2)*(I+',I4,')).')
C
 1000 CNR = D0
      DO 1035 I = 1, 250
 1035 SIGMA(I) = D0
      SSOBS = D0
      SUMSIG = D0
      SUMRST = D0
      IV1 = -1
      MAXC1 = 1
C
C INPUT OF CONSTANTS USED FOR THE FINAL SPECIFICATION OF THE DEGREE-VAR-
C IANCE MODEL. R = RATIO BETWEEN THE BJERHAMMAR SPHERE RADIUS AND THE
C MEAN EARTH RADIUS, VARDG2 = VARIANCE OF TKE GRAVITY ANOMALIES AND IMAX
C WHICH IS EQUAL TO THE ORDER OF THE LOCAL COVARIANCE FUNCTION USED.
C THUS IMAX=0 INDICATES, THAT WE ARE USING A GLOBAL COVARIANCE FUNCTION.
C IN THIS LAST CASE 4 SET OF EMPIRICAL ANOMALY DEGREE-VARIANCES OF ORDER
C UP TO IMAX0 CAN DE INPUT, THEESE DEGREE-VARIANCES WILL HAVE TO BE
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```
C GIVFN ON THE SURFACE OF THE MEAN EARTH, CF.REF(B),EQ.(PP).
      READ(5,101)R,VARDG2,IMAX
  101 FORMAT(F9.6,F7.2,I4)
      IF (R.GT.D1 .OR. VARDG2.LT.DO.OR.IMAX.LT.0) GO TO 9999
      LZERO = IMAX .NE. 0
      IMAXI = IMAX+1
      IF(LZERO)GO TO 1040
C INPUT OF THF FORMAT OF, THE HIGHEST DEGREE OF, AND THE EMPIRICAL ANO-
C KALY DEGREF-VARIANCES IN UYITS OF MGAL**2.
  102 FORMAT(I 2)
      READ(5,102) IMAXO
      IMAX = IMAXO
      IMAX1 = IMAX+1
      READ(S, 103) FMT
  103 FORMAT(9A8)
      READ(5,FMT) (SIGMA(I), I = 3, IMAX1)
C NOTE THAT THE DEGREE-VARIANCE OF ORDER I IS STORED IN SIGMA(I+1).
C
      DO 1001 I = 3, IMAX1
 1001 SUMSIG = SUMSIG + SIGMA(I)
 1040 IF (IMAX1+IS.LT.250) GO TO 1002
      WRITE(6,108)
  108 FORMAT(' SUBSCRIPTS OF ARRAY SIGMA EXCEEDS ARRAY LIMIT, STOP.')
      GO TO 9999
C
 1002 S = R*R
      S2 = S*S
      RL = (D1-R)*(D1+R)
      S1 = RL
      RLNL = DLOG(RL)
      IF (IMAX.LT.3) GO TO 1004
      RI = DFLOAT(IMAX)
      DO 1003 J = 3, IMAX
      GO TO (1101,1102,1103),KTYPE
 1101 RA = (RI-D1)/RI
      GO TO 1105
 1102 RA = (RI-D1)/(RI-D2)
      GO TO 1105
 1103 RA = (RI-D1)/((RI-D2)*(RI+IK))
 1105 SUMRST = (SUMRST+RA)*S
 1003 RI = RI- D1
C
      SUMRST = SUMRST*S2
 1004 DIF = VARDG2-SUMSIG
      IF (DIF.LT.DO) GO TO 9999
C COMPUTATION OF THE NORMALIZING CONSTANT A, CF. REF.(B) PARA. 3030
      GO TO (1106,1107,1108),KTYPE
 1106 RA = RLNL+S/RL-S2/D2
      GO TO 1110
 1107 RA = D1/RL-S2*RLNL-S2-S-DI
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      GO TO 1110
 1108 CALL SUMK(S,S2,RL,S1,D1,RA,W,PW,LT,LT)
      RA =(-S2*RLNL+IK1*RA)/IK2
 1110 A = DIF/(RA-SUMRST)
C
      IF (IMAX .LT. 3) GO TO 1006
      IF (LKEQ1) SIGMAO(3+IS) = SIGMA(3)-A*S2/D2
      SI = S2
      RI = D3
      RI1 = D2
      DO 1005 I = 4, IMAX1
      SI = SI*S
      GO TO (1111,1112,1113),KTYPE
 1111 RA = D1/(RI*RI1)
      GO TO 1115
 1112 RA = 1/(RI1*(RI-D2))
      GO TO 1115
 1113 RA = 1/(RI1*(RI-D2)*(RI+IK))
 1115 SIGMAO(I+IS) = SIGMA(I)/(RI1**2)-A*RA*SI
      RI1 = RI
 1005 RI = RI+D1
C THE DEG.VAR. OF THE COVARIANCE FUNCTION OF THE ANOMALOUS POTENTIAL
C ARE STORED IN THE FIRST PART OF SIGMA (SUBSCRIPT 1 TO IMAX1R) FOR COL-
C LOCATION I AND IN THE LAST PART (SUBSCRIPT IS=IMAX1R+3 TO IS+IMAX1)
C FOR COLLOCATION II.
C
 1006 U0 = D2-D1/DFLOAT(IMAX1+1)
C
  110 FORMAT('0RATIO R/RE                                  =        ',F9.6,/
     *' VARIANCE OF POINT GRAVITY ANOMALIES     = ',F10.2,' MGAL**2'/
     *' THE FACTOR A                            = ',F10.2,' MGAL**2')
      A0 = A/(S*S)
      WRITE(6,110)R,VARDG2,A0
  111 FORMAT(I3,' EMPIRICAL ANOMALY DEGREE-VARIANCES FOR DEGREE > 1,'/,
     *' (IN UNITS OF MGAL**2):')
C
      IF(LZERO)GO TO 1014
      WRITE(6,111)IMAX
      WRITE(6,FMT) (SIGMA(I), I = 3, IMAX1)
      GO TO 1015
C
 1014 WRITE(6,112)IMAX
  112 FORMAT( I4,' DEGREE-VARIANCES EQUAL TO ZERO')
C
C INPUT OF OBSERVATIONS OR COORDINATES OF PREDICTION POINTS.
 1015 WRITE(6,225)
  225 FORMAT('0 OBSERVATIONS:'/)
      IF(LNEQ)GO TO 2006
C
C INPUT OF LOGICALS DEFINING TYPE OF PREDICTION OUT-PUT. LGRID IS TRUE,
```

```
C WHEN PREDICTIONS SHALL BE MADF IN POINTS OF A UNIFORM GRID, LFRNO IS
C TRUE,WHEN THE ESTIMATED ERRPR OF PREDICTION SHALL PE COMPUTED. LCOMP
C IS TRUE, WHEN OBSERVED AND PREDICTED VALUES SHALL BE COMPAIRED.
 2000 READ(5,200)LGRID,LERNO,LCOMP
  200 FORMAT(3L2)
      IF (.NOT.(LERNO.AND.LKESOL)) GO TO 2002
      LERNO = LF
      WRITE(6,226)
  226 FORMAT(' *** ERROR WILL NOT BE COMPUTED, REQUIRED YFQ NOT STORED.
     ****')
 2002 LCOMP = LCOMP .AND.(.NOT.LGRID)
      IF (LCOM.OR.(.NOT.LCOMP)) GO TO 2005
      LCOM = LT
C INPUT OF SCALE FACTORS, USED FOR TABLE OF DISTRIBUTION OF DIFFERENCE,
C SEE SUBROUTINE COMPA.
      READ(5,203)VG,VF
      CALL COMPA(VG,VF)
C VG IN MGAL AND VF IN ARCSEC.
C
 2005 LNERNO = .NOT.LERNO
      LMAP = LF
      LMEGR = LF
      DM = D1
      DA = D0
      IF(.NOT.LGRID)GO TO 2006
C
C INPUT OF COORDINATES (LATITUDE,LONGITUDE IN DEGREES AND DEC. OF MINU-
C TES) OF SOUTH-WEST CORNER OF GRID, MAGYITUDE OF GYID INCREMENTS IN
C NORTHERN AND EASTERN DIRECTIOYS (MINUTES), NUMBER OF INCREMENTS IN
C THE SAME DIRECTIONS, THE VALUE OF IKP: (1 FOR ZETA, 2 FOR DELTA
C G, 3 FOR KSI, 4 FCR FTA AND 5 FOR (KSI,ETA)), RP = THE RATIO BETWEEN
C THE RADIUS OF THE SPHERE ON WHICH TYE POINTS OF PREDICTION ARE SITUA-
C TED AND RE, THE VALUE OF LMAP, WHICH IS TRUE, WHEN THE PREDICTIONS
C SHALL BE PRINTED AS A PRIMITIVE MAP? THE VALUE OF LPUNCH, WHICH IS
C TRUE WHEY THE PREDICTIONS SHALL BE PUNCHED AND  THE VALUE OF LMEAN,
C TRUE WHEN THE PREDICTED QUANTITIES ARE MEAN VALUES (BECAUSE THIS IM-
C PLIES? THAT THEY ARE REPRESENTED AS POINT VALUES IN A CERTAIN HEIGHT).
      READ(5,201)IDLAC,SLAC,IDLOC,SLOC,GLA,GLO,NLA,NLO,IKP,RP,LMAP,
     *LPUNCH,LMEAN
  201 FORMAT(2( I5,F6.2),2F7.2,2I4,I2,F10.7,3L2)
      IF (NLA.GT.19 .OR. NLO.GT.19 .OR. IKP.EQ.5) LMAP = .FALSE.
      LWLONG = LF
      LGRP = IKP.EQ.2
      NAI = 0
      NOI = 0
      IOBS2 = 0
      IH = 0
      H = D0
      IF (.NOT.LMEAN) H = (RP-D1)*RE
      LKM = H.GE.1.0D4
```

```
      HO = H
      OBS(1) = K
      IF (LKM) OBS(1) = H*1.0D-3
      LSTOP = LT
      IANG = 2
      NO = 0
C
 2001 SLAT = NAI*GLA+SLAC
      SLON = NOI*GLO+SLOC
      ISLA = SLAT/60+0.1D-2
      ISLO = SLON/60+0.1D-2
      IDLAT = IDLAC+ISLA
      IDLON = IDLOC+ISLO
      SLAT = SLAT-60*ISLA
      SLON = SLON-60*ISLO
      NE = NO+1
      H = HO
      IF (NOI .EQ. NLO) GO TO 2003
      NO1 = NOI+1
      GO TO 2004
 2003 NOI = 0
      NAI = NAI+1
 2004 IF(NAI.NE.0 .OR. NOI .NE. 1)GO TO 2031
      GO TO 2007
C
C INPUT OF ONE PATA-SET OF OBSERVATIONS OR COORDINATES OF PREDICTION
C POINTS. ALL RECORDS MUST BE PUNCHED IN THE SAME WAY, THERE ARE THE
C FOLLOWING RESTRICTIONS AND OPTIONS: A STATION NUMBER MAY BE USED, BUT
C IT MUST OCCUPY THE FIRST GATAFIELD ON THE RECORD. THE TWO NEXT DATA-
C FIELDS MUST CONTAIN THE GEODETIC LATITUDE AND LONGITUDE ( IN AN ARBI-
C TRARY ORDER). IN CASE THE HEIGHT IS GIVEN, MUST IT BE PUNCHED IN THE
C NEXT DATAFIELD. THE FOLLOWING UP TO FOUR DATPFIELDS WILL WAVE TO
C CONTAIN THE OBSERVED QUANTITY (OR QUANTITIES WHEN A PAIR OF DEFLECTI-
C ONS ARE OBSERVED) AND CONTINGENTLY THE STANDARD DEVIATIONS (WHEN LE IS
C TRUE AND LSA IS FALSE), A LIST OF COORDINATES OF POINTS WILL HAVE TO
C CONTAIN A HEIGHT OR A FICTICIOUS OBSERVATION. THE LAST DATAFIELD HAVE
C TO HOLD THE VALUE OF A LOGICAL VARIABLE LSTOP, TRUE FOR THE LAST
C RECORD IN THE FILE AND FALSE (I.E. BLANK) OTHERWISE.
C
C INPUT OF THE FORMAT OF THE RECORDS HOLDING THE OBSERVATION OR THE CO-
C ORDINATES OF THE PREDICTION POINT.
 2006 READ(5,103)FKT
C IYPUT OF VARIABLES SPECIFYIYG THE CONTENT OF THE RECORDS. INO = 1,
C WHEN THE STATION NUMBER IS PUNCHED, 0 OTHERWISE, ILA, ILO THE NUMBER
C OF THE DATAFIELDS OCCUPIED FY THE LATITUDE AND THE LONGITUDE RESPEC-
C TIVELY, IANG  SPECIFYING UNITS OF ANGLES (1 FOR DEGREES, MINUTES* ARC,-
C SECONDS, ? FOR DEGREES, MINUTES, 3 FOR DEGREES AND 4 FOR 400-GRADES),
C IH= THE NUMBER OF THE DATAFIELD HOLDING THE HEIGHT (ZERO WHEN NO
C HEIGHT IS CONTAINED), IOBS1, IOBS2 = THE DATAFIELD NUMBER OF THE FIRST
C AND THE SECOND OBSERVATION, RESPECTHVELY (ZERO WHEY NO FIRST OR SECOND
```

```
C OBSERVATION), IKP, SPECIFYIYG THE KIND OF OBSERVATION, (1 FOR ZETA, 2
C FOR MEASURED GRAVITY* POINT OR MEAN GRAVITY ANOMALIES, 3 FOR KSI, 4
C FOR ETA AND 5 FOR PAIR OF DEFLECTIONS (KSI,ETA) OR (ETA,KSI),(IN THE
C SAME ORDER AS THE LATITUDE AND THE LONGITUDE)).
C THEN THE RATIO RP (CF.REF(B), EQ.(87)) BETWEEN THE SPHERE ON WHICH THE
C OBSERVATIONS ARE SITUATED AYD RE, THE VALUES OF 5 LOGICAL VARIABLES:
C LPUNCH = PUNCH OBS. OR PREDICTED VALUE AND CONTINGENTLY THEIR DIFFE-
C RENCE* LWLONG = LONGITUDE IS POSITIVE TOWARDS WEST, LMEAN = THE DYED-
C ICTED OR OBSERVED QUANTITY IS A MEAN VALUE* LSA = ALL OBSERVED QUAN-
C TITIES HAVE THE SAME STANDARD DEVIATIONS AND LKM = THE HEIGHT IS IN
C UNITS OF KILOMETERS.
      READ(5,202)IND,ILA,ILO,IANG,IH,IOBS1,IOBS2,IKP,RP,LPUNCH,LWLONG
     *,LMEAN,LSA,LKM
  202 FORMAT(8I3,F10.7,5L2)
      GM = D1
      DA = DO
      LGRP = IKP.EQ.2
      IF (LGRP) READ(5,203)DM,DA,LMEGR
C LMEGR IS TRUE, WHEN THE MEASURED GRAVITY VALUE IS INPUT. DM AND DA ARE
C AN ADDITIVE AND A MULTIPLICATIVE CONSTANT* RESPECTIVELY, WHICH CAN BE
C USED TO CONVERT INPUT VALUES TO MGAL OR CORRECT FOR A SYSTEMATICAL
C ERROR.
  203 FORMAT(2F10.2,L2)
C INPUT OF STANDARD DEVIATLON.
      IF (LSA) READ(5,212)WM
  212 FORMAT(F6.2)
C
 2007 LREPEC = IKP.EQ.5
C INITIALIZATION OF VARIABLES IN COMMON BLOCK /PR/.
      LONECO = .NOT.LREPEC
      LNKSIP = IKP.NE.3 .AND. LONECO
      LNETAP = IKP.NE.4 .AND. LONECO
      LDEFVP = IKP .GT. 2
      LNDFP = .NOT.LDEFVP
      LNGR = .NOT.LGRP
C LREPEC IS TRUE, WHEN TWO COLUMNS CAN BE COHPUTED AT THE SAME TIME.
C LNSKIP, LNETAP IS TRUE, WHEN THE OBSERVATION OR REQUESTED PREDICTION
C IN P IS NOT KSI RESP. ETA.
      LZETA = IKP.EQ.1
      LKSIP = .NOT.LNKSIP
      IF (RP.LT.R) RP = D1
C CHECK? THAT POINT IS NOT INSIDE THE BJERHAMMAR-SPHERE.
C
C COMPUTATION OF CONSTANTS USED TO NORMALIZE THE NORMAL-EQUATIONS OR
C SCALE THE ESTIMATES OF THE ERRORS OF PREDICTION.
      NI = MAXC1
      P(JR) = D1
      P(JR+1) = RP
      INDEX(JR+1) = IKP
      PW = D1
```

```
      COSLAP = D1
      SINLAP = D0
      RLATP  = D0
      RLONGP= D0
      CALL PRED(S,SREF,U0,A,IS,1599,JR,1,1,IMAX1,LF,LF,LT)
      PW = C(MAXC1)
      IF (PW.GT.D0) PW = DSQRT(PW)
      P(JR) = PW
      W = WP(IKP)
      IF (LNGR) W = W*RP*RP
      IF (LZETA) W = W*RP
      PWO = PW*W
      PW2 = PWO*PWO
      IF (LCREF) JVL = -1
C
C OUTPUT OF HEACIYG AND INITIALIZATION OF VARIABLES.
      LINVDE = LONECO.OR.(IOBS2.EQ.0).OR.(IOBS1.LT.IOBS2)
      LOUTC = LNEQ.OR.LCOMP
      IF (LMEAN) WRITE(6,205)
  205 FORMAT('OTHE FOLLOWING QUANTITIES APE MEAN-VALUES, AND ARE REPRESE
     *NTED AS POINT VALUES IN A HEIGHT R.')
      IF (RP.GT.1.03D0.AND.(LKSIP.OR.LGRP).AND.(LTRAN.OR.LPOT).AND.
     *LPOTSD) WRITE(6,204)
  204 FORMAT('0** WARNING: THE HEIGHT MAY BE TCO BIG FOR THE COMPUTATION
     * OF',/,' THE REFERENCE GRAVITY OR THE CYANGE IN LATITUDE **')
C
      CALL HEAD(IKP,LONECO,PWO,RP)
C INITIALIZATION OF LOGICAL VARIABLES USED TO DETERMINE WHICH QUANTITIES
C WE WILL HAVE TO ADD TOGETHER TO FORM THE FINAL OUTPUT OR TO DETERMINE
C WHICH QUANTITIES WILL BE INPUT.
      LADRA = IB.NE.IA
      LADDBC = IB.NE.IC1
      LADDBP = IB.NE.IP
      LADBPR = LADDBP.AND.LREPEC
      LTYB = LTRAN.AND.(IT.NE.IB)
      LTEB = LTRAN.AND.(IT.EQ.IB)
      LOE1 = LE.AND.(.NOT.LSA).AND.LONECO
      LOE2 = LE.AND.(.NOT.LSA).AND.LREPEC
C K1 HAS PEEN INITIALIZED BY THE CALL OF 'HEAD'.  IT IS EQUAL TO THE NUM-
C BER OF QUANTITIES READ IN TO THE ARRAY OBS.
      IF (LOE1) K1 = K1+1
      IF (LOE2) K1 = K1+2
C
      IF (LGRID) GO TO 2031
C
      IF (LCOMP) CALL COMPB(IKP,LNGR,LREPEC,LONECO)
C
C INPUT OF COORDINATES OF OBSERVATION OR PREDICTION POINTS 4YD CONTIN-
C GENTLY THE OESERVED QUANTITIES AND THEIR STANDARD DEVIATIONS.
      IJ = IANG*2+INO-1
```

```
 2023 GO TO(2024,2025,2026,2027,2028,2029,2028,2029),IJ
 2024 READ(5,FMT)IDLAT,MLAT,SLAT,IDLON,MLON,SLON,(OBS(I),I=1,K1),LSTOP
      GOTO 2030
 2025 READ(5,FMT)NO,IDLAT,MLAT,SLAT,IDLON,MLON,SLON,(OBS(I),I=1,K1),
     *LSTOP
      GO TO 2030
 2026 READ(5,FMT)IDLAT,SLAT,IDLON,SLON,(OBS(I),I=1,K1),LSTOP
      GOTO  2030
 2027 READ(5,FMT)NO,IDLAT,SLAT,IDLON,SLON,(OBS(I),I=1,K1),LSTOP
      GOTO  2030
 2028 READ(5,FMT)SLAT,SLON,(OBS(I),I=1,K1),LSTOP
      GO TO 2030
 2029 READ(5,FMT)NO,SLAT,SLON,(OBS(I),I=1,K1),LSTOP
C
 2030 IF(ILA .LT. ILO)GO TO 2031
C CORRECTING IYVERTED ORDER OF LAT. AND LONG*
      I = IDLAT
      IDLAT = IDLON
      IDLON = I
      I = MLAT
      MLAT = MLON
      MLON = I
      AO = SLAT
      SLAT = SLON
      SLON = AO
C
 2031 CALL RAD(IDLAT,MLAT,SLAT,RLATP,IANG)
      IF (LWLONG.AND.IANG.LE.2) IDLON = -IDLON
      IF (LWLONG.AND.IANG.GT.2) SLON = -SLON
      CALL RAD(IDLON,MLON,SLON,RLONGP,IANG)
      COSLAP = DCOS(RLATP)
      SINLAP = DSIN(RLATP)
      IF (LGRID) GO TO 2049
C
      IF (LOE1) WOBS(N+1) = OBS(K1)
      IF (LOE2) WOBS(N+1) = OBS(K1-1)
      IF (LOE2) WOBS(N+2) = OBS(K1)
      IF (IH.NE.0) GO TO 2050
      IF (LREPEC) OBS(12) = OBS(2)
      OBS(2) = OBS(1)
      OBS(1) = DO
      GO TO 2051
 2050 OBS(12) = OBS(3)
      IF (LWLONG.AND.IKP.GT.3) OBS(12) = -OBS(12)
C CORRECTING THE OBSERVATION BY AN ADDITIVE AND MULTIPLICATIVE CONSTANT
C (IS USED ONLY FOR GRAVITY OBSERVATIONS).
 2051 OBS(2) = OBS(2)*DM+DA
      H = OBS(1)
      IF (LKM) H = H*1.0D3
C CONVERSION OF HEIGHT INTO METERS.
```

```
 2049 IF (LNGR) GO TO 2056
C
      IF (H.GT.25.0D3.AND.(.NOT.LPOTSD))CALL EUCLID(COSLAP,SINLAP,
     *RLONGP,H,E21,AX1)
      CALL RGRAV(SINLAP,H,0,GREF)
C COMPUTING THE GRAVITY ANOMALY.
      IF (LMEGR) OBS(2) = OBS(2)-GREF
C
 2056 IF (LINVDE) GO TO 2032
      OB1 = OBS(12)
      OBS(12) = OBS(2)
      OBS(2) = OB1
      IF (.NOT.LE.OR.LSA) GO TO 2032
      WM = WOBS(N+1)
      WOBS(N+1) = WOBS(N+2)
      WOBS(N+2) = WM
C
 2032 OBS(IB) = DO
      IF (LREPEC) OBS(IB1) = DO
      IF (LNTP) GO TO 2055
C
      IF (LNTRAN) GO TO 2053
      CALL EUCLID(COSLAP,SINLAP,RLONGP,DO,E21,AX1)
      CALL TRANS(SINLAP,COSLAP,RLATP,RLONGP,IKP,IT)
      IF (LNGR) GO TO 2053
      GREF1 = GREF
      IF (H.GE.25.0D3) CPLL EUCLID(COSLAP,SINLAP,RLONGP,H,E22,AX2)
      CALL RGRAV(SINLAP,H,15,GREF)
      OBS(IT) = GREF1-GREF
      IF (LPOTSD) OBS(IT) = OBS(IT)-13.7D0
      GREF1 = GREF
C
 2053 IF (LNPOT) GO TO 2055
      IF (LNGR) GO TO 2054
C
C FOR GRAVITY ANOMALIES WE USE, THAT THE NEW REF. POTENTIAL GIVES A BET-
C TER ESTIMATE OF THE GEOMETRIC HEIGHT, SO THAT THE NEW REF. GRAVITY CAN
C BE COMPUTED AT THIS HEIGHT,
      CALL EUCLID(COSLAP,SINLAP,RLONGP,DO,E22,AX2)
      CALL SPOT(RLATP,COSLAP,RLONGP,1,11,GREF,UREFO,DO)
      H = H+OBS(11)
 2054 CPLL EUCLID(COSLAP,SINLAP,RLONGP,H,E22,AX2)
      IF (LZETA) CALL UREFER(UREF,15,H)
      IF (LKSIP) CALL CLAT(CLA,15,H,RLATP)
      CALL GPOT(RLATP,COSLAP,RLONGP,IKP,IP,GREF,UREF,CLA)
      IF (LADDBP) OBS(IB) = OBS(IB)+OBS(IP)
      IF (LADBPR) OBS(IB1) = OBS(IB1)+OBS(IP1)
 2055 IF (.NOT.LCREF) GO TO 2052
C
C THE VARIABLE IV1 IS USED TO INDICATE, THAT THE DEGREE-VARIANCES STORED
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C IN SIGMA WILL HAVE TO BE CHANGED FPPM COLLOCATION I TO COLLOCATION II.
C THE VALUE IS TRANSFERRED TO PRED BY THE COMMON BLOCK /PR/.
      CALL PRED(SR,SREFR,URO,AR,0,0,2,IOBSR,NIR,IMAX1R,LT,LF,LF)
      IV1 = -1
      OBS(IC1) = PREDP*W
      IF (LADDBC) OBS(IB) = OBS(IB)+OBS(IC1)
      IF (LONECO) GO TP 2052
      OBS(IC11) = PRETAP*W
      IF (LADDBC) OBS(IB1)= OBS(IB1)+OBS(IC11)
 2052 IF (LPRED) GO TO 3021
C
C STORING COORDINATES AND RIGHT-HAND SIDE OF NORMAL-EQ., N COUNTS THE
C COLUMNS AND IC THE STATIONS.
      N = N+1
      IC = IC+1
    . IF (LTNB) OBS(IU) = OBS(IB)-OBS(IT)
      IF (LTEB) OBS(IU) =-OBS(IT)
      IF (LK30) OBS(3) = OBS(2)-OBS(IU)
      OB1 = OBS(K2)/PWO
      B(N) = OB1
      IF (LSA) WOBS(N) = WM
      SSOBS = SSOBS+OB1**2
      IF (LONECO) GO TC 2033
C
      IF (LTNB) OBS(IU1) = OBS(IB1)-OBS(IT1)
      IF (LTEB) OBS(IU1) = -OBS(IT1)
      IF (LK30) OBS(13) = OBS(12)-OBS(IU1)
      OB2 = OBS(K21)/PWO
      SSOBS = SSOBS+OB2**2
      N = N+1
      B(N) = OB2
      IF (LSA) WOBS(N) = WM
 2033 IF (N.LE.1598) GO TO 2060
C  CHECK OF NUMBER OF OBSERVATIONS NOT EXCEEDING ARRAY LIMIT
      WRITE(6,229)
  229 FORMAT(' NCMEER OF OBSERVATIONS TOO BIG. *** STOP ***')
C
 2060 COSLAT(IC) = COSLAP
      SINLAT(IC) = SINLAP
      RLAT(IC) = RLATP
      RLONG(IC) = RLONGP
      CNR =RLATP*IC+CNR
C OUTPUT OF OBSERVATIONS.
      CALL OUT(NO,IDLAT,MLAT,SLAT,IDLON,MLON,SLON,LONECO)
      IF(.NOT.LSTOP)GO TO 2023
C
  230 FORMATT(3L2)
      READ(5,230)LSTOP,LRESOL,LWRSOL
C
      IF (LRESOL.AND.LWRSOL) GO TO 9999
```

```
C
C ESTABLISHING A CATALOGUE OF THE OBSERVATIONS,MAXIMALLY 9 SETS ALLOWED.
      INDEX(JR) = IC+1
      JR = JR+2
      IF (INDEX(JR-3) .NE. IKP .OR. (DABS(P(JR-3)-RP).GT.1.0D-7))
     *GO TO 2034
      JR = JP-2
      INDEX(JP,-I) = INDEX(JR+1)
 2034 IF ((JR-II) .LT. 19) GO TO 2035
      WRITE(6,298)
  298 FORMAT(' OBSERVATIONS ARRANGED TOO COMPLICATED')
      GO TO 9999
 2035 HF (.NOT.LSTOP) GO TO 2006
C
C END OF INPUT CF OBSERVATIONS. M = NUMBER OF OBSERVATIONS, IOBS =
C NUMBER CF OBSERVATION POINTS.
      IOBS = IC-ISO
      N = N-ISO
      N1 = N+1
      B(N1+ISO) = SSOBS
C
      IF (.NOT.LF) CO TO 2036
      WRITE(6,297)(WOBS(I+ISO), I = 1, N)
  297 FORMAT('OSTANDARD DEVIATIONS OF THE OBSERVATIONS IN THE SAME SEQUE
     *NCE'/,' AND IN THE SAME UYITS AS THE OBSERVATIONS:'/,(' ',10F8.2))
C
 2036 IF (LRESOL) GO TO 3228
C
      IF (LDEFF) GO TO 2037
      LDEFF = LT
      NT = 3
      IDIMC = 4700
C IN THIS PROGRAM-VERSION, THE ARRAY C HAS DIMENSION IDEYC AYD ITS
C VALUES ARE STORED OR RETRIVED FPOM UNIT 8 BY NT READ OR WRITE OPERA-
C TIONS.
C
C SETTING UP A CATALOGUE OF THE NORMAL EQUATIONS
C NB IS THE RECORD NUMBER, IC COUNTS THE NUMBER OF COLUMNS WITHIN A
C RECORD, AND THIS NUMBER IS STORED IN NCAT(100). YCAT (I) WILL CONTAIN
C THE SUBSCRIPT OF THE DIAGONAL ELEMENT OF COLUMN I-1, AND ALL THE
C ELEMENTS OF ISZE WILL BE ZERO BECAUSE WE WORK WITH A FULL MATRIX,
C NBL(I) CONTAINS THE NUMBER OF THE LAST COLUMN IN RECORD 1-1. NOTE
C THAT MAXIMALLY 98 COLUMNS MAY BE STORED IN ONE RECORD,
 2037 NBT = I
      NBL(1)=0
      N5 = 1
      IC=0
      I1=0
C
      DO 2304 I = 1, N1
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      IC=IC+1
      ISZE(IC)=0
      NCAT(IC)=I1
      H1 = I1+I
      I2=I1+I+1
C
      IF((I2.LE.IDIMC).AND.(I.NE.N1).AND.(IC.LE.98)) GO TO 2304
      NCATB IC+1) = I1
      IF(I.NE.N1) GO TO 2303
      I2=I2+N1+1
      IF(I2.LE.IDIMC) GO TO 2301
C SECURING THAT THF LAST COLLUMN + ONE MORE CAN BE STORED IN THF SAME
C RECORD.
      NCAT(100)=IC-1
      WRITE(8'NBT+2)C3,NCAT,ISZE
      NBT = NBT+NT
      NB=NB+1
      NBL(NB)=I-1
      NCAT(2)=N1
      I1=N1
      IC=1
C MAXC IS THE SUBSCRIPT OF THE DIAGONAL ELEMENT OF THE CAST COLUMN,  AND
C MAXC2 IS THF SUBSCRIPT PF THE FICTICIOUS DIAGONAL FLEMENT OF THE RIGHT
C HPND SIDE (IN WHICH THE SQUARE-SUM OF THE NORMALIZED OBSERVATIONS IS
C STORED).
 2301 MAXC = I1-N1
      MAXC2 = I1
C STORING THF RIGHT- HAND SIDE.
      DO 2307 J=1,N1
 2302 C(MAXC+J) = B(J+ISO)
C
 2303 I1=0
      NCAT(100)=IC
      IQ = NET
      WRITE(8'IQ)C,NCAT,ISZE
      NBT = NBT+NT
      IC=0
      NB=NB+1
      NBL(NB)=I
      IF (NB.LE.309) GO TO 2304
      WRITE(6,299)
  299 FORMAT(' RESERVED AREA PN FILE 8 TOO SMALL')
      GO TO 9999
 2304 CONTINUE
C
      MAXBL=NB-1
      MAXPLT = (MAXBL-1)*NT+1
C
C COMPUTATION OF ELEMENTS OF NORMAL EQUATIONS (EQUAL TO THE COVARIANCE
C BETWEEN THE OBSERVATIONS). THE COEFFICIENTS ARE STORED IN THE ONE-DI-
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C MENSIONAL ARRAY C, COLLUMN AFTER COLLUMN,THE DIAGONAL ELEMENT HAVING
C THE HEIGHEST SUBSCRIPT.
C
C INITIALIZING VARIABLES:
C NI IS IN MAIN THE SUBSCRIPT OF THE FIRST ELEMENT OF COLUMN NC I ARRAY
C C. IN THE SUBROUTINE PRED, NI IS THE SUBSCRIPT OF THE ELEMENTS OF THE
C COLUMN, NB  IS THE NUMBER OF THE BLOCK IN WHICH THE COVARIANCES ARE
C STORED AND I1 IS TEE NUMBER OF THE LAST CPLUMQ STORED IN THE BLOCK.
C ICNEXT IS THE NUMBER OF THE FIRST COLUMN WITHIN A GROUP OF DATA WITH
C THE SAME CHARACTERISTICS. (THE CHARACTERISTICS ARE GIVEN BY THE ARRAYS
C INDEX AYD P (SUBSCRIPTS JC AND JC+1)).
      NI = 1
      NC = 1
      JC = II
      IV1 =-1
      NB=1
      I1=NBL(2)
      READ(8'NT)C3,NCAT,ISZE
      FIND(8'1)
      NBT = I
      ICNEXT = ISO+1
C
      DO 3100 IC = 1, IOBS
      ICC = IC+ISO
      COSLAP = COSLAT(ICC)
      SINLAP = SINLAT(ICC)
      RLATP = RLAT(ICC9
      RLONGP = RLONG(ICC)
C
      IF(ICC .NE. ICNEXT)GO TO 3003
      IKP = INDEX(JC+1)
C INITIALIZATION OF VARIABLES IN COMMON BLOCK /PR/.
      PW = P(JC)
      RP = P(JC+1)
      ICNEXT = INDEX(JC)
      JC = JC+2
      LREPEC = TKP .EQ. 5
      LONECO = .NOT.LREPEC
      LGRP = IKP.EQ.2
      LNGR = .NOT.LGRP
      LNKSIP = IKP.NE.3 .AND. LONECO
      LNETAP = IKP.NE.4 .AND. LONECQ
      LDEFVP = IKP .GT. 2
      LNDFP = .NOT.LDEFVP
C
      PWO = PW*WP(IKP)
      IF (LNGR) PWO = PWO*RP*RP
      IF (IKP.EQ.1) PWO = PWO*RP
 3003 LBST=LREPEC.AND.(NC.EQ.I1)
C AS WE FOR LREPEC=TRUE ARE COMPUTING TWO COLUMNS AT THE SAME TIME, WE
```

```
C MUST, IN CASE THE SFCOND CCLUYN IS THE FIRST ONE IN THE NFXT RECORD
C STORE THIS ONE TEMPORARY IN ARRAY B. THE PROBLEM WILL ONLY OCCUR WHEN
C WE ARE SETTING UP THE NORMALEQUATIONS. LBST = B-STORE*
C
      CALL PRED(S,SREF,UO,A,IS,ISO,II,IC,NC,IMAX1,LF,LBST,LT)
C
      ND = NI-1
      DIA = C(ND)
      IF (LE) C(ND) = DIA+(WOBS(NR-1)/PWO)**2
      IF (LONECO) GO TO 3020
      IF (LE) DIA = DIA+(WOBS(NR)/PWO)**2
      IF (LEST) B(NR) = DIA
      IF (.NOT.LBST) C(NI+NC) = DIA
C THE PPECEDING STATEMENT ASSURES, THAT THE DIAGONAL ELFYENT CORRESPON-
C DING TO ETAP BECOMES EQUAL TO THAT OF KSIP.
      NC = NC+1
      NI = NI+NC
C
 3020 IF (NC.LT.I1.AND.NC.LT.N) GO TO 3100
C
C STORING THE COEFFICIENTS OF THE NORMAL-EQ. ON FILE 8, RECORD NB.
      IO = NBT
      WRITE(8'IQ)C,NCAT,ISZE
      IF (.NOT.LONEQ) GP TO 3200
C
C OUTPUT OF COEFFICIENTS OF NORMAL-EQUATIONS,
      WRITE(6,380)NB
  380 FORMAT('OCOEFFICIENTS OF NORMAL-EQUATIONS* BLOCK ',I4,/)
      I1 = NI- I
      IF (NB.EQ.MAXBL) I1 = MAXC2
C
      WRITE(6,381)(C(K), K = 1, I1)
  381 FORMAT(' ',10F8.4)
 3200 NBT = NBT+NT
      NB=NB+1
      NI=1
      IF (NC.NE.N) 11 = NBL(NB+1)
C
      IF (NB.GT.MAXBL) GO TO 3201
      READ(8'IQ)C1
      READ(8'IQ)C2
      READ(8'IQ)C3,NCAT,ISZE
C WE HAVE TO READ THE WHOLE CONTENT OF BLOCK NB INTO ARRAY C, RFCAUSE
C WE MUST BE SUPE THAT THE RIGHT-HAND SIDE (WHICH ALREADY IS STORED)
C IS PLACED CORRECTLY.
      FIND(8'NB1)
C
 3201 IF (.NOT.LBST) GO TO 3100
C
      DO 3202 K=1,NC
```

```
 3202 C(K) = B(K+ISO)
      NI = NC+1
 3100 NC = NC+1
C END OF LOOP FORMING NORMAL-EQUATIONS.
C
      CALL NES(N1,0,0,.TRUE.,PW)
C
C PUNCHING: NUMBER OF OBSERVATION POINTS, NUMBER OF OBSERVATIONS, DIF-
C FERENCE BETWEEN SQUARESUM OF OBSERVATIONS AND NORM OF APPROXIMATION,
C A CHECK-NUMBER (KEE) CNR, AND FINALLY THE SOLUTIONS AND THE SQUARE-SUM
C OF THE OBSERVATIONS.
      IF (LWRSOL)WRITE(7,361)IOBS,N1,PW,CNR,(C(J+MAXC), J = 1, N1)
  361 FORMAT(2I5,2D15.7,/,(4D20.13))
      GO TO 3229
C
C INPUT OF SOLUTIONS.
 3228 MAXC = 0
      READ(5,361)I,N1C,PW,CNRC,(C(J+MAXC),J = 1, N1)
C CHECK OF SOLUTIONS CORRESPOND TO OBSERVATIONS.
      IF (IOBS.EQ.I.AND.N1.EQ.N1C.AND.DABS(CNRC-CNR).LT.0.1D0)GO TO 3229
      WRITE(6,354)
  354 FORMAT(' SOLUTIONS DO NOT CORRESPOND TO INPUT DATA, STOP.')
      GO TO 9999
 3229 WRITE(6,300)
  300 FORMAT('0SOLUTIONS TO NORMAL EQUATIONS:'/)
      WRITE(6,301)(C(J+MAXC), J = 1, N)
  301 FORMAT(1X,5D17.10)
      IF (LRESOL) WRITE(6,362)
  362 FORMAT('0THE SOLUTIONS HAVE BEEN COMPUTED IN A PREVIOUS RUN.')
C
      MAXC2 = MAXC+N1
      WRITE(6,353)N,C(MAXC2),PW
  353 FORMAT('0NUMBER OF EQUATIONS =',I4,/
     *' NORMALIZED SQUARE-SUM OF OBSERVATIONS          =',D13.6,/,
     *' NORMALIZED DIFFERENCE BETWEEN SQUARE-SUM OF'/
       OBSERVATIONS AND NORM OF APPROXIMATION          =',D13.6,/)
C
C STORING SOLUTIONS DIVEDED BY A COMMON FACTOR.
      NI = P
      JRNEXT = ISO+1
      JR = II
      DO 3032 I = 1, IOBS
      IF ((I+ISO).NE.JRNEXT) GO TO 3030
      IKP = INDEX(JR+1)
      LONECO = IKP .LT. 5
      JRNEXT = INDEX(JR)
      PW = P(JR)
      JR = JR+2
 4030 IF (LONECO) GO TO 303%
      B(NI+ISO) = C(MAXC+NI)/PW
```

```
      NI = NI+1
 3031 B(NI+ISO) = C(MAXC+NI)/PW
 3032 NI = NI+1
      IF (LC1) GO TO 3110
C
      LC1 = LT
C INPUT OF LCREF, WHICH IS TRUE WHEN ONE MORE SET OF OBSERVATIONS
C SHALL BE INPUT AND USED FOR THE ESTIMATION OF ONE MORE HARMONIC FUNC-
C TION.
      READ(5,230)LCPFF
      IF (.NOT.LCRFF) GO TO 3110
C
C STORING AWAY THE NECESSARY CONSTANTS FOR COLLOCATION I.
      SR = S
      SREFR = S
      URO = UO
      IOBSR = IOBS
      AR = P
      IMAX1R = IMAX1
      NIR = N1
C INITIALIZING VARIABLES FOR STAR1 OF COLLOCATION II.
      IS = IMAX+3
      II = 22
      JR = 22
      IC = NIR+2
      N = IC
      ISO = IC
      INDEX(21) = IC
      WRITF(6,345)
  345 FORMAT('OSTART OF COLLOCATION II:'/)
      GO TO 1000
C
C INITEALIZIYG VARIABLES FOP. PREDICTION. MAXC1 IS THE SUBSCRIPT OF THE
C FIRST ELEMENT IN THE COLUMN FORKING THE RIGHT-HAND SIDE,
 3110 LPRED = LT
      LNEQ = LF
      LE = LF
      LC2 = LCREF
      MAXC1 = MAXC+1
      INDEX(41) = 0
      JR = 41
      WRITE(6,344)
  344 FORMAT('1 PREDICTIONS:'/)
      GO TO 2000
C
 3021 NI = MAXCI
      CALL PRED(S,SREF,UO,A,IS,ISO,II,IOBS,N1,IMAX1,LT,LF,LERNO)
      IF (LNFRNO) GO TO 3022
C
      C(MAXC2) = D1
```

```
C STORING THE NEW RIGHT-HAND SIDE ON FILE 8, SO THAT THE ERROR OF PRE-
C DICTION CAN BE COMPUTED.
      IQ = MAXBLT
      WRITE(8'IQ)C,NCAT,ISZE
C COMPUTATION OF ERROR OF PREDICTION.
      CALL NES(N1,N,0,.FALSE.,OBS(K2))
      OBS(K2) = OBS(K2)*PW2+0.1D-6
      IF (OBS(K2) .GT. 0.1D-9) OBS(K2) = DSQRT(OBS(K2))
C
 3022 IF (LONECO) GO TO 3026
      IF (LNERNO) GO TO 3024
      FIND(8'MAXBLT)
      DO 3025 J = 1, N
 3025 C(MAXC+J) = C(MAXC2+J)
      C(MAXC2) = D1
      IQ = MAXBLT
      WRITE(8'IQ)C,NCAT,ISZE
      CALL NES(N1,N,0,.FALSE.,OBS(K21))
      OBS(K21) = OBS(K21)*PW2+0.1D-6
      IF (OBS(K21).GT.0.1D-9) OBS(K21) = DSQRT(OBS(K21))
 3024 OBS(IA1) = PRETAP*W
      IF (LADBA) OBS(IB1) = OBS(IB1)+OBS(IA1)
      IF (LTRAN) OBS(IU1) = OBS(IB1)-OBS(IT1)
 3026 OBS(IA) = PREDP*W
      IF (LADBA) OBS(IB) = OBS(IB)+OBS(IA)
      IF (LTRAN) OBS(IU) = OBS(IB)-OBS(IT)
C
      IF (LCOMP) CALL COMPC(IU)
C
      CALL OUT(NO,IDLAT,MLAT,SLAT,IDLON,MLON,SLON,LONECO)
      IF (LMAP) IMAP(NO) = OBS(IU)*100
      IF(LGRID .AND. NAI .LE. NLA) GO TO 2001
C
      IF (.NOT.LMAP) GO TO 3045
C
      K = NLA+1
      WRITE(6,360)IDLAC,SLAC,IDLOC,SLOC,GLA,GLO
  360 FORMAT('1MAP OF PREDICTIONS:'/' COORDINATES OF SOUTH-WEST CORNER'/
     *'  LATITUDE  LONGITUDE AND SIDE LENGTH LAT.   LONG,  '/
     *'     D  M      D  M      M        M'/2(I4,F6.2),2F8.2,//)
      DO  3044 J = 1, K
      WRITE(6,350)
  350 FORMAT('0 ')
      NAP = NO-NLO
      WRITE(6,352)(IMAP(I), I = NAP, NO)
  352 FORMAT(' ',20I5)
 3044 NO = NAP-1
C
 3045 IF (.NOT.LSTOP) GO TO 2023
C
```

```
      READ(5,230)LSTOP
      IF (.NOT.LSTOP) GO TO 2000
C
      IF (LCOM) CALL OUTCOM
C
 9999 STOP
      END


      SUBROUTINE NES(NN,IIFC,IIFR,LBS,PW)
C THE SUBROUTINE WILL? USING THE CHOLESKYS METHOD:
C (1) COMPUTE THE REDUCED MATRIX L CORRESPONDING TO A SYMMETRIC POSITIVE
C     DEFINITE (NN-1)*(NN-1) MATRIX A, WHEN THE IIFC COLUMNS AND IIFR
C     ROWS OF L ARE KNOWN, (L*LT = A, LT THE TRANSPOSED OF L).
C (2) COMPUTE THE REDUCED NN-1 VECTOR (L**-1)*Y.
C (3) COMPUTE THE DIFFFRENCE PW = YN-YT*(A**-1)*Y.
C (4) SGLVE THE ECVATIOYS LT*X = (L**-1)*Y,(THE SO CALLED BACK-SOLUTION)
C
C THE REDUCEG ROWS AND COLUMNS OF L (THERE MAY BE NONE), THF CORRES-
C PONDING UNREDUCED UPPER TRIANGULAR PART OF A, THE NN-VECTOR FORMED
C BY Y AND YN FORMS AN UPPER TRIANGULAR NN*NN-MATRIX.
C THE MATRIX IS STORED COLUMNVIZE IN NT*MAXBL RECORDS OF A DIRECT ACCESS
C FILE (UNIT NUMBER 8). THE YT RECORDS (NT*12800 BYTES) CONTAINS AS MANY
C COLUMNS AS POSSIBLE IN THE FIRST 8*(NT-1)*1600+8*1500 BYTES CORRESPON-
C DING TO THE DIMENSION OF THF ARRAYS C AND CR. THE LAST 800 BYTES
C OF THE NT RECORDS HOLDS TWO CPTALOGES (INTEGER ARRAYS) NCAT AND ISZE
C (NRCAT, IRSZE RESPECTIVELY). WE WILL CALL THE NT RECORDS A BLOCK.
C WHEN THE CONTENT OF A BLOCK HAS BEEN TRANSFERRED E.G. TO C, NCAT,
C ISZE, WE HAVE THF FOLLOWING SITUATION. THE COLUMNS ARE STORED IN C
C FROM THE FIRST ELEMENT DIFFERENT FROM ZERC TC THE DIAGONAL ELEMENT.
C NCAT(I) IS THE SUESCRIPT CF THE DIAGONAL FLEMEWT OF COLUMN I-1 AND
C ISZE(I) IS THF NUMBER OF IGNORED (SAVED) ZEROES IN COLUMN I. YCAT(100)
C IS THE NUMBER OF CDLUMNS STORED IN THE RECORD.
C NBL(I) IS EQUAL TO THE NUMBER OF THF LAST COLUMN STORED IN RECORD I-1.
C (1) TO (3) ABOWE WILL ALWAYS PE EXECUTED, BUT (4) WILL ONLY BE EXE-
C CUTED WHEN THE LOGICAL LBS IS TRUE* THE EXECUTION OF (1), (2) FUD (4)
C IS EQUIVALEYT TO THF SOLUTION OF THE EQUATIONS A*X=Y. THE SOLUTIONS
C WILL BE STORED IN THE ARRAY C IN THE POSITIONS ORIGIYALLY OCCUPIED BY
C Y AND WILL BE TRANSFERRED TO MAIN THROUGH THE COMMON BLOCK.
C IN CASE A NUMERICAL SINGULARITY CCCURS IN COLUMN NUMBER JD, THE COLUMN
C IS DELETED BY CHANGING THF CATALOGUE ISZE AND THE ELEMENTS IN ROW JD
C AND THE JD'TH ELEMENT OF THE SOLUTION VECTOR IS PUT EQUAL TO ZERO.
      IMPLICIT INTEGER(I,J,K,M,N), REAL *8(A-H,O-Z),LOGICAL(L)
      COMMON /NESOL/C(4700),NCAT(100),ISZE(100),NBL(310),MAXBL,ID
      COMMON /CPW/CR1(1600)
      DIMENSION CR(4700),NRCAT(100),IRSZE(100)
      EQUIVALENCE (CR(1),CR1(1))
      NT = 3
C NOTE, THAT IY CASE YT IS CHANGED, WILL IT ALSO BE YECESSAPY TO CHANGE
```

```
C THE DIMENSION OF THE ARRAYS C AND CR.
C
      N = NN
      IFR=IIFR+1
      IFC=IIFC+1
      IF (IFR.GT.IFC) WRITE(6,10)
   10 FORMAT(' ERROR IN CALL, IFR.GT.IFC .')
C
C REDUCTION OF COLUMNS IFC TO N. ELEMENTS, WHICH ARE ALREADY REDUCED,
C ARE STORED IN CR [EXCEPT FOR JBL=KBL). ELEMENTS9 WHICH ARE GOING TO BF
C REDUCED, ARE STORED IN C.
C
C FIND FIRST ACTUAL RECORD AND ROW/COLUMN.
      JBF=0
  200 JBF=JBF+1
      IF(NBL(JBF+1).LT.IFC) GO TO 200
      IL = NBL(JBF)
      JF = IFC-IL
      JTF = JBF*NT-NT+1
      JTL = JTF
      ID = JTF
      FIND(8'ID)
C
      KBF = 0
  201 K6F = KBF+1
      IF (NBL(KBF+1).LT.IFR) GO TO 201
      KLO = NBL(KBF)
      KF = IFR-KLO
      KFO = KF
      KTF = (KBF-1)*NT+1
C
C READ RECORD JPL FROM FILE.  THE ARRAY C WILL CONTAIN AT LEAST ONE UNRE-
C DUCED COLUMN.
      DO 280 JBL=JBF,MAXBL
      READ(8'ID)C,NCAT,ISZE
      FIND(8'KTF)
      NC=NCAT(100)
      JO=NBL(JBL)
      KL = KLO
      ID = KTF
C
      DO 270 KBL = KBF, JBL
      LREC=KBL.EQ.JBL
      READ(8'ID)CR,NRCAT,IRSZE
      NR=NRCAT(100)
      KO=KL
      KL=KL+NR
      MR=MAXO(KO,IIFR)
C
C IN ORDER TO MINIMIZE THE NUMBER OF TRANSPORTS TO AND FROM FILE 8, WE
```

```
C DO NOT (GENERALLY) COMPUTE PLL THE REDUCED ELEMENTS OF ONE COLUMN, BUT
C ONLY THE ELEMENTS IN ROW KO+1 TO KL. (KO IS THE NUMBER OF THE LAST
C COLUMY IN THE PREVIOUS RECORD* KL THE NUMBER OF THF LAST IN THE ACTUAL
C RECORD. )
C
      DO 260 J=JF,NC
      ISZ=ISZE(J)
C WE CHECH THAT THFRE ARE ELEMENTS (UNREDUCED) DIFFERENT FROM ZERO IN
C COLUMN J WITH SUBSCRIPT GREATHER THAN OR FQUAL TO KL.
      IF (KL.LE.ISZ) GO TO 260
C THE SUBSCRIPT OF THE ELEMENT IN COLUMN J,ROW K IS NCAT(J)+K-ISZF(J).
C THE DIFFERENCE NCAT(J)-ISZE(J) IS STORED IN THE VARIABLE ICO.
C THE SAME DIFFERENCF FOR COLUMN (ROW) K IS STORED IN IRO. THE ELEMENT
C JUST BEFORE THE FIRST ELEMEYT TD BE REDUCED WILL HENCE HAVE SUBSCRIPT
C I=ICO+MAX(KO,IIFR,ISZ). KG IS THE ABSOLUTE ROW NUMBER-
      ICO = NCAT(J)-ISZ
      JD=JO+J
      NRO=MINO(JD,KL)-KO
      IF (MR.GE.ISZ) KFS = KF
      IF (MR.LT.ISZ) KFS = ISZ-KO+1
C
      DO 250 K = KFS, NRO
      KD = KO+K
      K1 = KD-1
      I = ICO+KD
C I IS THE SUBSCRIPT OF THE COEFFICIENT TO BE REDUCED.
      QSUM = 0.0D0
      IRSZ=IRSZE(K)
      IRO = NRCAT(K)-IRSZ
      QCI=C(I)
      KF1=MAXO(IIFR,ISZ,IRSZ)+1
C IRSZ = KD INDICATES THAT COLUMN KD CONTAINS A NUMFRICAL SINGULARITY.
C THE ELEMENTS OF RCW KD IS PUT EGUAL TO ZERO.
      IF (IRSZ .FQ. KD) QCI = QSUM
C
      IF(KF1.GT.K1) GO TO 245
      IF (LREC) GO TO 235
C REDUCTION OF ONE COEFFICIENT.
      DO 230 M = KF1, K1
  230 QSUM = QSUM+C(ICO+M)*CR(IRO+M)
      GO TO 245
  235 DO 240 M = KF1, K1
  240 QSUM = QSUM+C(IRO+M)*C(ICO+M)
  245 QC1 = QCI-QSUM
C
      IF(LREC) GO TO 246
      C(I) = QCI/CR(IRO+KD)
      GO TO 250
  246 IF (JD.NE.KD) C(I) = QCI/C(IRO+KD)
  250 CONTINUE
```

```
C
      IF(.NOT.LREC.OR.(JD.NE.KD).OR.JD.EQ.N) GO TO 260
G
C  TEST OF NUMERICAL STABILITY
      QCI2 = QCI/C(I)**2
      IF (QCI2 .GT. 0.1D-16) GO TO 251
      WRITE(6,20)JD,QCI2
   20 FORMAT(' NUMERICAL SINGULARITY IN ROW NO.',I5,', TEST QUANTITY =',
     *D17.10)
      ISZE(J)=JD
C THE COLUMN IS DELETED.
      GO TO 260
C
  251 C(I) = DSQRT(QCI)
  260 KF = 1
  270 CONTINUE
C
C REDUCED ARRAY C BACK TO FILE. FIRST COLUMN IN NEXT RECORD IS NOW THE
C FIRST STORED, BUT FIRST REDUCED COLUMN IS AGAIN COLUMN KFO IN REC. JBF
      ID = JTL
      WRITE(8'ID)C,NCAT,ISZE
      JTL = JTL+NT
      JF=1
      KF=KFO
C
      IF(JBL.NE.MAXBL) GO TO 280
      PW = QCI
      IF(.NOT.LBS) GC TO 280
C
C BACK-SOLUTION. NOTE THAT THE VARIABLE I AT THIS MOMENT IS THE SUB-
G SCRIPT OF THE DIAGONAL ELEMENT OF THE COLUMN CONTAINING THE RIGHT- HAYO
C SIDE.  I WILL SUCCESSIVELY TAKE THE VALUE OF THE SUBSCRIPT OF THE ELE-
C MENT IN RCJW M OF THIS COLUMN,  AND THE SOLUTION WILL BE STORED IN C(I).
      M = N
      KTL = (MAXBL-1)*NT+1
      ID = KTL
C
      DO 277 KB= 1,MAXBL
      READ(8'ID)CR,NRCAT,IRSZE
      KTL = KTL-NT
      ID = KTL
      IF (KTL.GT.0) FIND(8'ID)
      NR=NRCAT(100)
C THE COLUMN CONTAINING THE RIGHT-HAND SIDE IS SKIPPED,
      IF(KB.EQ.1)NR=NR-1
      K1=NR+1
      IF(NR.EQ.0) GO TO 277
      DO 276 K=1,NR
C WE STEP BACKWARD (M) FROM COLUMN N-1, TAKING THE NR COLUMNS FROM EACH
C RECORD SUCCESSIVELY.
```

```
      I=I-1
      IC=I
      M=M-1
      IR=NRCAT(K1)
      K1=K1-1
      IRSZ=IRSZF(K1)
      MR=M-IRSZ
      IF (MR.GT.0) GO TO 373
C IN CASE A COLUKN HAS BEEN DELETED, THE UNKNOWN IS PUT EQUAL TO ZERO*
      C(I) = 0.0D0
      GO TO 276
C THE (UN)KNOWN C(I) IS COMPUTED (ONE EQUATION WITH ONE UNKNOWN).
  273 CI = C(I)/CR(IR)
      C(I) = C1
C
      IF(MR.EQ.1) GO TO 276
      DO 274 MP=2,MR
C IR IS THE SUBSCRIPT OF THE ELEMENTS OF COLUMN M, RUNNING FROM THE
C DIAGONAL-1 UP TO THE SUBSCRIPT OF THE FIRST ELEMENT DIFFERENT FROM
C ZERO. IC IS THE SUBSCRIPT OF THE ELEMEYTS ON THF RIGHT-HAND SIDE IN
C THE SAME ROW AS CR(IR).
      IR=IR-1
      IC=IC-1
C THE CONTRIBUTION FROM THE (UN)KNOWN C(I) IS SUBTRACTED FROM THE QUAN-
C TITIES ON THE RIGHT-HAND SIDE.
  274 C(IC) = C(IC)-CI*CR(IR)
C
  276 CONTINUE
  277 CONTINUE
C THE SOLUTIONS ARE NOW ALL STORED IN C FROM C(NCAT(N)+1) TO C(NCAT(N+1)
C -1). THEY ARE TRANSFERRED TO 'MAIN'  THROUGH THE COMMON BLOCK NESOL.
  280 CONTIYUE
      RETUQN
      END


      SUBROUTINE ITRAN(DX,DY,DZ,EPS1,EPS2,EPS3,DL,AX2,E22,RLATO,
     *RLONGO,DKSIO,DETAO,DZETAO,LCHANG)
C  TRANSFORMATION OF GEODETIC COORDINATES AND CORRESPONDING DEFLECTIONS
C OF THE VERTICAL AND HEIGHT ANOMALY, WHEN THF COORDINATE SYSTEM IS
C TRANSLATE9 BY (DX,DY,DZ), ROTATED (EPS1,EPS2,EPS3) AROUND THE X,Y,Z
C AXES RESPECTIVELY AND MULTIPLIED BY 1+DL.
C  WHFN LCHAYG IS TRUE, THERE IS FUTHERMOEE A CHAYGF IN THE ASTRONOMICAL
C  COORDINATES AND IN THF HEIGHT SYSTEM, GIVEN AS A CHANGE OF THF DE-
C  FLFCTIONS OF THF VERTICAL AND THE HEIGHT ANPMALY IN A POINT HAVING
C  COORDINATFS RLATO AND RLONGO.
      IMPLICIT INTEGER(I), REAL *8(A-H,O-Z), LOGICAL(L)
      COMMON /OBSER/OBS(20)
      COMMON /EUCL/X,Y,Z,XY,XY2,DISTO,DIST2
```

```
      DO = 0.0D0
      D1 = 1.0D0
      RADSEC = 206264.806D0
      EPS1 = EPS1/RADSEC
      EPS2 = EPS2/RADSEC
      EPS3 = EPS3/RADSEC
      IF (.NOT.LCHANG) GO TO 60
      SINLAO = DSIN(RLATO)
      COSLAO = DCOS(RLATO)
   60 RETURN
C
      ENTRY TRANS(SINLAP,COSLAP,RLATP,RLONGP,IKP,IT)
C INPUT OF COS AND SIN TO LATITUDE, LATITUDE, LONGITUDE (RADIANS),
C IKP SIGNIFYING WHICH KIND OF CHANGE IN THE OBSERVATIONS WE WANT TO
C COMPUTE AND IT EQUAL TO THE SUBSCRIPT IN THE ARRAY OBS IN WHICH THE
C  RESULT IS RETURNED.
      IT1 = IT
      IF (IKP.EQ.5) IT1 = IT+10
      DKSI = DO
      DETA = DO
      DZETA = DO
      S1 = D1+DL
      X1 = X
      Y1 = Y
      X = DX+S1*(X+EPS1*Y-EPS2*Z)
      Y = DY+S1*(Y-EPS1*X1+EPS3*Z)
      Z = DZ+S1*(Z+EPS2*X1-EPS3*Y1)
      XY2 = X*X+Y*Y
      XY = DSQRT(XY2)
      DIST2 = XY2+Z*Z
      DISTO = DSQRT(DIST2)
      RLONG = DATAN2(Y,X)
C
C  COMPUTATION OF THE NEW GEODETIC LATITUDE, CF REF(C) PAGE 183.
      S = AX2/DSQRT(D1-E22*SINLAP**2)
      DH = DO
      RLAT1 = RLATP
   70 RLAT = RLAT1
      RLAT1 = DATAN2(Z,XY-E22*S*COSLAP)
      COSLAP = DCOS(RLAT1)
      SINLAP = DSIN(RLAT1)
      S = AX2/DSQRT(D1-E22*SINLAP**2)
      DH = XY/COSLAP-S
      S1 = (RLAT1-RLAT)*2/(RLAT1+RLAT)
      IF (DABS(S1).GT. 0.1D-8) GO TO 70
      DLO = (RLONG-RLONGP)*RADSEC
      DLA = (RLAT1-RLATP)*RADSEC
      RLONGP = RLONG
      RLATP = RLAT1
      IF (.NOT.LCHANG) GO TO 69
```

```
C
C   CF. REF(C) PAGE 208, FORMULA (5-59)
        COSDLO = DCOS(RLONGP-RLONGO)
        SINDLO = DSIN(RLONGP-RLONGO)
        GO TO (61,75,62,63,62),IKP
    61 DZETA = (-(COSLAO*S IULk P-SINLAO*COSLAP*COSDLO)*DKSIO-COSLAP*SINDLO
      **DETAO)*AX2/RADSEC+(SINLAO*SINLAP+COSLAO*COSLAP*COSDLO)*DZETAO
        GO TO 69
    62 DKSI = (COSLAP*COSLAO+SINLAP*SINLAO*COSDLO)*DKSIO-SINLAP*SINDLO*
      *DETAO-(SINLAO*COSLAP-COSLAO*SINLAP*COSDLO)*RADSEC*DZETAO/AX2
        IF (IKP.NE.5) GO TO 69
    63 DETA = SINLAO*SINDLO*DETAO+COSDLO*DETAO+COSLAO*SINDLO*RADSEC
      **DZETAO/AX2
C
    69 GO TO (71,75,72,73,72),IKP
    71 OBS(IT) = DZETA+DH
        GO TO 75
    72 OBS(IT) = DKSI-DLA
        IF (IKP.NE.5) GO TO 75
    73 OBS(IT1) = DETA-DLO*COSLAP
    75 RETURN
        END


        SUBROUTINE GRAVC(AX,F,GM,I,LPOTSD,UREF,GAMMA)
C THE SUBROUTINE COMPUTES FIRST BY TuE CALL OF GRAVC THE CONSTANTS TO EE
C USED IN THE FORMULA F09 THE YDRMAL GRAVITY, THE FORMULA F04 THE NORMAL
C POTENTIAL AND THF CHANGE IN LATITUDE WITH YFIGHT. CDYSTAQTS RFLATED TO
C TWO DIFFERENT REFERENCE FIELDS MAY BE USED. THEY ARE STORED IN THE
C ARRAY FG IN THE VARIABLES SUBSCRIPTED FROM 1 TO 15 FOR TYF FIRST
C FIELD AND FRCM 16 TO 30 FOP THE SECOND ONF. THE ARRAY FJ CONTAINS THE
C ZOYAL HARMONICS, WITH SIGN OPPOSITE TO THE USUAL CONVENTIONS, CF
C REF(C), EQ. (2-92).
        IMPLICIT INTEGER(I), REAL *8(A-H,M-Z), LOGICAL(L)
        COMMON /DCON/DO,D1,D2,D3
        COMMON /EUCL/X,Y,Z,XY,XY2,DISTO,DIST2
        DIMENSION FG(30),FJ(30),LP(2)
        D4 = 4.0D0
        D5 = 5.0D0
        OMEGA2 = (0.7292115150-4)**2
        LP(1+I/15) = .NOT.LPOTSD
        E2 = F*(D2-F)
        AX2 = AX*AX
        FG(I+14) = AX
        FG(I+15) = GM
C
        IF (LPOTSD) GO TO 1501
        DO 1550 J = 1, 15
  1550 FJ(J+I) = DO
```

```
      FJ(I+1) = D1
      E = DSQRT(E2)*AX
      EM2 = E2/(D1-E2)
      BX2 = AX2*(D1-E2)
      BX = DSQRT(BX2)
      M = OMEGA2*AX2*BX/GM
      F2 = F*F
      FM = F*M
      TA = DATAN2(E,BX)
      E3 = E2*AX2
C QO IS THE QUANTITY REF(C), EQ.(2-58).
      QO = ((D1+D3*BX2/E3)*TA-D3*BX/E)/D2
      DO 1551 K = 1, 5
      K2 = 2*K
      DK = DFLOAT(K)
      DK2 = D2*DK
 1551 FJ(I+K2+1) = (-1)**K*D3*E2**K*(D1-DK+D5*DK*(D1-D2/15.0D0*M*E/(BX
     * *QO))/D3)/((DK2+1)*(DK2+D3))
C
C GAMMA IS THE NORMAL GRAVITY AT EUQATOR, CF. REF(C), EQ. (2-105A) AND
C (2-70). THE FIVE FOLLOWING COEFFICIENTS ARE FOUND IN REF(C) EQ.
C (2-115) AND (2-124).
      GAMMA =(GM/(AX*BX)-(D3/D2+D3*EM2/14)*OMEGA2*AX)*1.0D5
      FG(I+1) = GAMMA
      FG(I+2) = -F+D5*M/D2+F2/D2-26*FM/7+15*M*M/D4
      FG(I+4) = (-F2+D5*FM)/D2
      FG(I+3) = -D2*GAMMA*(D1+F+M)/AX
      FG(I+5) = D2*GAMMA*(D3*F-D5*M/D2)/AX
      FG(I+6) = D3*GAMMA/AX2
C CF. REF(C), EQ. (2-118),(2-119).
      FG(I+11) = (D2*F-M-F2)/D3+D2*FM/21.0D0
      FG(I+12) = -D4*F2/D5+D4*FM/7.0D0
C CF. REF(C), EQ. (2.61).
      UREF = GM*TA/E+OMEGA2*AX2/D3
      GO TO 1502
C
 1501 GAMMA = 978049.0D0
      UREF = 62639787.0D0
      FG(1) = GAMMA
C
C CONSTANTS USED IN INTERNATIONAL GRAVITY FORMULA, CF.REF(C),(2-126),
C (2-131) AND (2-128).
      FG(4) = 4*0.0000059
      FG(2) = 0.0052884-FG(4)
      FG(3) = -0.3087772400
      FG(5) = 0.0004520600
      FG(6) = 7.265D-8
C
 1502 FG(I+13) = (FG(I+2)+FG(I+4))*6.47512D-2
      FG(I+7) = UREF
```

```
C FG(I+8) CONTAINS THE THIRD DERIVATIVE OF THE NORMAL GRAVITY.
      FG(I+8) = D4*FG(I+6)/(AX*D3)
      RETURN
C
      ENTRY RGRAV(SINLAP,H,I,GREF)
      IF (H.GT.25.0D3 .AND. LP(1+I/15)) GO TO 1504
C  COMPUTATION OF THE REFEPENCE GRAVITY IN UNITS OF MGAL, CF. REF.(C),
C  PAGE 77 AND 79. H MUST PE IN UNITS OF METERS.
      SIN2 = SINLAP*SINLAP
      GREF = FG(I+1)*(D1+FG(I+2)*SIN2+FG(I+4)*SIN2*SIN2)
     * +(FG(I+3)+FG(I+5)*SIN2+(FG(I+6)-FG(I+8)*H)*H)*H
      RETURN
C
      ENTRY CLAT(CLA,I,H,RLATP)
      LKSI = .TRUE.
      IF (H.GT.25.0D3 .AND. LP(1+I/15)) GO TO 1504
C COMPUTATION OF THE CHANGE IN THE DIRECTION OF THE GRADIENT WITH HEIGHT
C CF. REF(C) EQ. (5-34).
      CLA =-FG(I+13)*H*(Z*XY)/DIST2
      LKSI = .FALSE.
      RETURN
C
      ENTRY UREFER(UREF,I,H)
C COMPUTATION OF THE VALUE OF THE NORMAL POTENTIAL* CF. REF(C), EQ.
C (6-14) AND (6-15).
      LZETA = .TRUE.
      IF (DABS(H).GT.1.0D-3) GO TO 1503
      UREF = FG(I+7)
      RFTUQN
C
 1504 LZETA = .FALSE.
C
 1503 T = Z/DISTO
      U = XY/DISTO
      AX = FG(I+14)
      GM = FG(I+15)
      A1 = DO
      AO = DO
      B1 = DO
      BO = DO
      DA1 = DO
      DAO = DO
      S = AX/DISTO
      TS = T*S
      S2 = S*S
      K = 11
      C1 = 12.0DO
      CO = 11.0DO
C
C SUMMATION OF LEGENDRE-SERIES REPRESENTING THE NORMAL POTENTIAL. BO
```

```
C WILL HOLD THE DERIVATIVE WITH RESPECT TO THE DISTANCE FROM THE ORIGIN
C AND DAO THE DERIVATIVE WITH RESPECT TO THE LATITUDE AFTER THE FINAL
C RECURSION STEP.
      DO 1553 J = PP 11
      FJK = FJ(K+I)
      C3 = D2-D1/CO
      C4 = CO*S2/C1
      A2 = A1
      A1 = AO
      AO = C3*TS*A1-C4*A2+FJK
      IF (LZETA) GO TO 1555
C
      DA2 = DA1
      DA1 = DAO
      DAO = C3*(S*A1+TS*DA1)-C4*DA2
      R2 = B1
      BP = 50
      BO = C3*TS*B1-C4*B2+FJK*CO
 1555 C1 = CO
      CO = CO-D1
 1553 K = K- I
C
      IF (LZETA) GO TO P554
C GP IS THE DERIVATIVE OF THE NORMAL GSAVITY WITH RESPECT TO DISTO
C (THE DISTANCE FROM THE ORIGIN) AYD GL IS THE DERIVATIVE WITH RESPECT
C TO THE LATITUDE, CF. REF(C) EQ. (6-20') AND (6-22).
      GL = (GM*DAO/DIST2-OMEGA2*DISTO*T)*U
      GP = -GM*BO/DIST2+OMEGA2*DISTO*U*U
      GZ = T*GR+U*GL
      GREF = DSQRT(GR*GR+GL*GL)
      IF (LKSI) CLA =-(DARSIN(-GZ/GREF)-RLATP)*206264.806D0
      LKSI = .FALSE.
      GREF = GREF*1.0D5
      RETURN
 1554 UREF = GM*AO/DISTO+OMEGA2*XY2/D2
      RETURN
      END


      SUBROUTINE IGPOT(GM,AX,COFF,M,NMAX)
C THE SUBROUTINE COMPUTES THE VALUE OF THE GRAVITATIONAL POTENTIAL AND
C THE THREE FIRST ORDER DERIVATIVES IN A POINT P HAVING GEODETIC COOR-
C DINATES RLATP,RLONGP AND EUCLIDIAY COORDINATES X,Y,Z. THE POTENTIAL IS
C REPRESENTED BY A SERIES IN QUASI-NORMALIZED SPHERICAL HARMONICS,HAVING
C COEFFICIEUTS OF HIGHEST DEGREE NMAX, ALL STORED IN THE ARRAY COFF.
C THE ALGORITHM USED IS DESCRIBED IN REF(D).
      IMPLICIT INTEGER(I,J,K,M,N), LOGICAL(L), REAL *8(A-H,O-Z)
      COMMON /EUCL/X,Y,Z,XY,XY2,DISTO,DIST2
      COMMON/OBSER/OBS(20)
```

```
      DIMENSION  F(55),G(55),A(25),B(25),COFF(M)
C
C THE ARRAYS P  AND B ARE USED TO HOLD THE VALVES OF THE SOLID SPHERICAL
C HARMONICS OF DIFFERENT DEGREES,  THE SUBSCRIPT OF A OR B IS EQUAL TO
C THE ORDER-1.
C THE ARRAY COFF CONTAINS THF FULLY NORMALIZED COEFFICIENTS OF THE POT-
C ENTIAL DEVELOPED IN A SERIES OF NORMALIZED SOLID SPHERICAL HARMONICS
C UP TO AND INCLUSIVE DEGREE NMAX AND FOUR DUMMY COEFFICIENTS EQUAL TO
C ZERO. THE COEFFICIENTS WILL BE QUAZI-NORMALIZED BY THE SUBROUTINE.
C F AND G CONTAINS SQUARE-ROOT TABLES USED IN THE RECURSION PRO-
C CEDURE. GM IS THE PRODUCT OF THE GRAVITATIONAL CONSTANT AND THE MASS
C OF THE EARTH (METERS**3/SEC**2), AX THE SEMI-MAJOR AXIS (METERS),
C OMEGA2 THE SQUARE OF THE SPEED OF ROTATION.
C INITIALIZING CONSTANTS.
      OMEGA2=(0.7292115150-4)**2
      RADSEC = 206264.806D0
      D0=0.0D0
      D1=1.0D0
      D2 = 2.0D0
      NMAX1=NMAX+1
      N3=NMAX+3
      N4 = 2*NMAX+3
C
C THE ZERO ORDER COEFFICIENT IS PUT EQUAL TO ZERO AND THE CONTRIBUTION
C FROM THIS TERM IS FIRST USED, WHEY THE CONTRIBUTION FROM ALL OTHER
C DEGREES HAS BEEN ACCUMULATED.
      COFF(1) = D0
C
C SETTING UP A SQUARE-ROOT TABLE.
      F(1) = D0
      G(1) = PO
      DO 1031 N = 1, N4
      DN = DFLOAT(N)
      F(N+1) = DSQRT(DN*(DN-D1))
 1031 G(N+1) = DSQRT(DN)
C
C THE COEFFICIENTS ARE GOOING TO BE QUASI- NORMALIZED.
      IJ = 1
      DO 1033 I = 1, NMAX
      DN = G(2*(I+1))*1.0D-6
      N2 = 2*I+1
      DO 1032 J = 1, V?
 1032 COFF(IJ+J) = COFF(IJ+J)*DN
 1033 IJ = IJ+N2
      SQ2=G(3)
      RETURN
C
      EYTRY GPOT(RLATP,COSLAP,RLONGP,IKP,IP,GREF,UREF,CLA)
C THF SUBROUTINE IS HERE USED TO COMPUTF HEIGHT-ANOMALIES, GRAVITY-AND-
C MALIES AYD DEFLECTIOYS (KSI, ETA OR (KSI,ETA)), CORRESPONDING TO THF
```

```
C VALUE OF IKP = 1,..,5. THE RESULT IS STORED IN OBS(IP) AYD OBS(IP+10)
C FOR IKP = 5.
C
      IP1 = IP
      IF (IKP.EQ.5) IPB = IP+10
      U=X/DISTO
      V=Y/DISTO
      W=Z/DISTO
      ADIVR=AX/DISTO
      POT-DO
      DX=DO
      DY=DO
      DZ=DO
C A(1) IS NOW THE VALUE OF THE SOLID SPH.HARMONIC OF DEGREE 0.
      A(1)=D1
      B(1) = CO
      FACT- PI
      ADIVRI=D1
C
      DO 7001 I = 2, N3
      A(I)=DO
 7001 B(I)=DO
C
      DO 7010 I=1,NMAX1
      C1 = D1
      A1=DO
      B1 = DO
      B2 = DO
      DA1=DO
      DB1=DO
      DB2=DO
      DXO=DO
      DYO=DO
      DZO=DO
      POTO=DO
      C = DO
      IS=(I-1)*(I-1)+1
C
      C2 = D1
      CO = SQ2
      A2=A(1)
      DA2=COFF(IS1
      IPLUSJ=I
      IPLUS1=I+1
      IMINJ1=I+2
      IS=IS+1
      ARFAC=ADIVRI/FACT
C
      DO 7005 J=1,IPLUS1
      IPLUSJ=IPLUSJ+1
```

```
      IMINJ1=IMINJ1-1
      JPLUS1=J+1
      ALFA = F(IPLUSJ)*C1
      ALFA? = ALFA*C2
      BETA = F(IMINJ1)*CO
      GAMMA=G(IMINJ1)*G( IPLUSJ)
C
      POTO=POTO+A2*DA2+B2*DB2
C
      A0=A1
      A1=A2
      A2=A(JPLUS1)
      B0=B1
      B1=B2
      B2=B(JPLUS1)
      ALAO=ALFA2*A0
      BEA2=BETA*A2
      ALBO=ALFA*B0
      BEB2=BETA*B2
      AJ=(U*(ALAO-BEA2)-V*(ALBO+BEB2))/D2+GAMMA*A1*W
      BJ=(U*(ALBO-BEB2)+V*(ALAO+BEA2))/D2+GAMMA*B1*W
      A(J)=AJ
      B(J)=BJ
C
      DA0=DA1
      DA1=DA2
      DC2 = COFF(IS)
      DB0=DB1
      DB1=DB2
      DB2 = COFF(IS+1)
      ALBO=ALFA*DB0
      BEB2=BETA*DB2
      ALAO=ALFA2*DA0
      BEA2=BETA*DA2
      DXO=DXO+(ALAO-BEA2)*AJ+(ALBO-BEB2)*BJ
      DYO=DYO+(ALAO+BEA2)*BJ-(ALBO+BEB2)*AJ
      DZO=DZO+GAMMA*(DA1*AJ+DB1*BJ)
      IS=IS+2
      C1 =D1/CO
      C2 = D2-C
      CO = D1
 7005 C = D1
C
      POT=POT+ARFAC*POTO
      FACT=FACT*I
      ARFAC=ADIVRI/FACT
C DX,DY,DZ IS THF VALUE OF THE FIRST DERIVATIVES OF THE PQTENTIAL WITU
C RESPECT TO X,Y,Z.
      DX=DX-ARFAC*DXO
      DY=DY-ARFAC*DYO
```

```
      DZ=DZ-ARFAC*DZO
C
 7010 ADIVRI=ADIVRI*ADIVR
C
C CONTRIBUTION FROM ROTATIONAL POTENTIAL.
      ROTX=OMEGA2*X
      ROTY = OMEGA2*Y
      ROTPOT = OMEGA2/D2*XY2
      POT = (D1+POT)*GM/DISTO+ROTPOT
      AO=GM/DIST2
      DX = (U-DX/D2)*AO-ROTX
      DY = (V-DY/D2)*AO-ROTY
      DZ = (W-DZ)*AO
      IF (IKP.LE.2) GP = DSQRT(DX*DX+DY*DY+DZ*DZ)
C GP IS THE GPAVITY IN P.
      CO TO (7006,7007,7008,7009,7008),IKP
 7006 OBS(IP) = (POT-UREF)/GP
      RETURN
 7007 OBS(IP) = GP*1.0D5-GREF
      PETCQN
 7008 OBS(IP) = (DATAN2(DZ,DSQRT(DX*DX+DY*DY))-RLATP)*RADSEC+CLA
      IF (IKP.LT.4) RETURN
 7009 OBS(IP1) = (DATAN2(GY,DX)-RLONGP)*RADSEC*COSLAP
      RETURN
      END


      SUBROUTINE EUCLID(COSLAP,SINLAP,RLONG,H,E2,AX)
C COMPUTATION OF EUCLIDIAN COORDINATES X,Y,Z , PISTANCF AND SQUARE OF
C DISTANCE FROM Z- AXIS XY, XY? AND DISTANCF AND SQUARE OF DISTANCE FROM
C THE ORIGIN DISTO AND DIST2 FROM GRODETIC COORDINATES REFERRING TO AN
C ELLIPSOID PAVING SEMI-MAJOR AXIS EQUAL TO AX AND SECOND EXCENTRICITY
C E2.
      IMPLICIT INTEGER(I,J,K,M,N), PEAL *8(A-H,O-Z)
      COMMON /EUCL/X,Y,Z,XY,XY2,DISTO,DIST2
      DN = AX/DSQRT(1.0D0-E2*SINLAP**2)
      Z = ((1.0D0-E2)*DN+H)*SINLAP
      XY = (DN+H)*COSLAP
      XY2 = XY*XY
      DIST2 = XY2+Z*Z
      DISTO = DSQRT(DIST2)
      X = XY*DCOS(RLONG)
      Y = XY*DSIN(RLONG)
      RETURY
      END
```

```
      SUBROUTINE RAD(IDEG,MIN,SEC,RA,IANG)
C THE SUBROUTINE CONVERTS FOR IANG = 1,2,3,4 ANGLES IN (1) DEGREES, MI-
C NUTEST SECONDS, (2) DEGREES, MINUTES, (3) DECREES AND (4) 400-DEGREES
C TO RADIANS.
      IMPLICIT INTEGER(I,J,K,M,N),REAL *8(A-H,O-Z)
      1 = 1
      IF (IDEG .LT. 0 .AND. IANG .LT. 3) 1 = -1
      GO TO (1,2,3,4),IANG
    1 SE =I*IDEG*3600+MIN*60+SEC
      GO TO 5
    2 SE=I*IDEG*3600+SEC*60
      GO TO 5
    3 SE = SEC*3600
      GO TO 5
    4 SE = SEC*3240
    5 RA= I*SE/206264.806D0
      RETUF?N
      END


      SUBROUTINE HEAD(IKP,LONECO,PWO,RP)
C OUTPUT OF HEADINGS AND INITIALIZATION OF THE FOLLOWING VARIABLES: IA,
C IB,IP,IT,I1,IA1,IB1,IP1,IT1,I21,I31,IC1,IC11 (ALL SUBSCRIPTS OF DIF-
C FERENT OUTPUT QUANTITIES), KZ - K4 (SUBSCRIPT BOUNDARIES FOR OUTPUT
C QUANTITIES),  K1 = UPPER LIMIT FOR QUANTITIES READ INTO OBS.
      IMPLICIT LOGICAL(L)
      COMMON/OUTC/K2,K3,K4,IU,K21,IU1,IANG,LPUNCH,LOUTC,LNTRAN,LNERNO
     *,LK30
      COMMON /CHEAD/IA,IB,IH,IP,IT,IA1,IB1,IP1,IT1,IC1,IC11,K1,IOBS1,
     *IOBS2,LPOT,LC1,LC?,LCREF,LKM
      LTRAN = .NOT.LNTRAN
      LERNO = .NOT.LNERNO
      K1 = 0
      IF (IOBS1.NE.0) K1 = 1
      GO TO (2008~2009~2010~2011~2012),IKP
 2008 WRITE(6,204)
  204 FORMAT('0   NO       LATITUDE      LONGITUDE       H     ZETA (M)')
      GO TO 2013
 2009 WRITE(6,205)
  205 FORMAT('0   NO       LATITUDE      LONGITUDE       H    DELTA G (MGAL)'
     *)
      GO TO 2013
 2010 WRITE(6,206)
  206 FORMAT('0  NO       LATITUDE      LONGITUDE       H    KSI (ARCSEC)')
      GO TO 2013
 2011 WRITE(6,207)
  207 FORMAT('0   NO       LATITUDE      LONGITUDE       H     ETA (ARCSEC)')
      GO TO 2013
 2012 WRITE(6,208)
  208 FORMAT('0   NO       LATITUDE      LONGITUDE       U    KSI/ETA (ARCSE
```

```
     *C)')
      IF (IOBS2 .NE. 0) K1 = 2
 2013 IF (IH.NE.0) K1 = K1+1
      WRITE(6,267)PWO,RP
  267 FORMAT('+',59X,' ST.VAR. =',F6.2,', RATIO R/RE =',F10.7,',')
C
      GO TO (2018,2019,2020,2021),IANG
 2018 WRITE(6,209)
  209 FORMAT('              D   M   S        D   M   S      M')
      GO TO 2022
 2019 WRITE(6,210)
  210 FORMAT('                  D   M            D   M      M')
      GO TO 2022
 2020 WRITE(6,211)
  211 FORMAT('             DEGREES       DEGREES        M')
      GO TO 2022
 2021 WRITE(6,212)
  212 FORMAT('             GRADES        GRADES         M')
C
 2022 IF (LKM) WRITE(6,213)
  213 FORMAT('+',37X,'K')
C
C WE NOW COYPUTE THE SUBSCRIPT OF THE DIFFFQFYT QUANTITIES, WHICH WILL
C BE STORED FOP LATER OUTPUT IN THE ARRAY OBS. THE DIFFERENCE BETWEEN
C THE OBSERVATION GIVEN IN THE ORIGINAL AND THE NEW REF.SYSTEM IN
C OBS(IT), THE CONTRIBUTION TO THE REF.POT. FROM THE HARMONIC EXPANSION
C IN OBS(IP), THE CONTRIBUTION FROM COLL.I IN OBS(IC1) AND FROM COLL.II
C IN OBS(IC2).
      IC1 = 11
      IF (LTRAN) GO TO 2105
      IF(LPOT) GO TO 2102
      IF (LC1) GO TO 2201
      IB = 4
      GO TO 2104
 2201 IC1 = 5
      IF (LC2) GO TC 2101
      IB=5
      WRITE(6,250)
  250 FORMAT('+',64X,' PRED')
      GO TO 2104
C
 2101 IB=7
      IC2=6
      WRITE(6,251)
  251 FORMAT('+',63X,' COLL1  COLL2   PRED')
      GO TO 2104
C
 2102 IP=5
      IF (LC1) GO TO 2202
      IB = 5
```

```
      WRITE(6,245)
  245 FORMAT('+',64X,' POT ')
      GO TO 2104
C
 2202 IC1 = 6
      IF (LC2) GO TO 2103
      IB=7
      WRITE(6,252)
  252 FORMAT('+',64X,' POT     COLL    PRED')
      GO TO 2104
C
 2103 IC2=7
      IB=8
      WRITE(6,253)
  253 FORMAT('+',64X,' POT   COLL1  COLL2  PRED')
 2104 K3 = IB-4
      IU = IB
      GO TO 2110
C
 2105 IT=5
      IF(LPOT) C3 70 2107
      IF (LC1) GO TO 2205
      IB = 3
      WRITE(6,246)
  246 FORMAT('+',64X,' TRP ')
      GO TO 2103
C
 2205 IC1 = 6
      IF (LC2) GO TO 2106
      IB=7
      WRITE(6,254)
  254 FORMAT('+',63X,'  TRA     PRED PRED-TRA')
      GQ TO 2109
C
 2106 IC2=7
      IB=8
      WRITE(6,255)
  255 FORMAT('+',63X,'  TRA   COLL1   COLL2  PRED   PRED-TRA')
      GQ TO 2109
C
 2107 IP=6
      IF (LC1) GO TO 2208
      IB = 6
      WRITE(6,247)
  247 FORMAT('+',63X,'  TPA    POT    POT-TRA')
      GO TO 2109
C
 2208 IC1 = 7
      IF (LC2) GO TC 2108
      IB=8
```

```
      WRITE(6,256)
  256 FORMAT('+',63X,'  TRA     POT     COLL    PRED   PRED-TRA')
      GO TO 2109
C
 2108 IC2=8
      IB=9
      WRITE(6,257)
  257 FORMAT('+',63X,'  TRA     POT   COLL1  COLL2   PRED PRED-TRA')
 2109 K3=IB-3
      IU = IB+1
C
 2110 IF (LC2) IA = IC2
      IF (.NOT.LCREF) IA = IC1
C
      IF (LOUTC) GO TO 2125
      IF(LERNO) GO TO 2112
      K2=1
      GO TO 3135
C
 2112 K2=2
      WRITE(6,260)
  260 FORMAT('+',43X,'  ERR')
      GO TO 2135
 2125 IF(K3.GT.0) GP TO 2127
      K2=2
      WRITE(6,261)
  261 FORMAT('+',43X,'  OBS')
      GO TO 2135
C
 2127 IF (LERNO) GO TO 7128
      K2=3
      WRITE(6,262)
  262 FORMAT('+',43X,'  OBS     DIF')
      GO TO 2135
C
 2128 K2=4
      WRITE(6,263)
  263 FORMAT('+',43X,'  OBS     DIF     ERR')
C
 2135 K4 = K2
      LK30 = K3.GT.0
      IF (LONECO) GO TO 2111
      IB1 = IB+10
      IU1 = IU+10
      IA1 = IA+10
      IT1 = IT+10
      IP1 = IP+10
      IC11 = IC1+10
      K4 = 2*KZ-1
      K21 = K2+10
```

```
C
 2111 RETUQY
      END


      SUBROUTINE OUT(NO,IDLAT,MLAT,SLAT, IDLON,MLON,SLON,LONC)
C THE SUBROUTINE WRITES ON UNIT 6 (1) STATION NUMBER,(2) COORDINATES,
C (3) DESERVED VALUE (IN ORIG.REF.SYSTEM),(4) DIFFERENCE BETWEEN OBSER-
C VED AND PREDICTED VALUE, (5) ERROR OF PREDICTION, (6) TRANSFORMATION
C VALUE, (7) SPHERICAL HARMONIC SERIES CONTRIBUTION, (8) RESULT OF COLL
C I AND (9) COLL.II, (10) SUM OF QUANTITIES (7)-(9) AND (11) SUM OF (6)-
C (9) - ALL IF MEANINGFULL. IN CASE WE ARE DEALING WITH A PAIR OF DE-
C FLECTIONS, (LONC = FALSE), THE CORRESPONDING QUANTITES FOR ETA ARE
C WRITTEN A LIVE BELOW.
C WHEN LPUNCH IS TRUE, THE FOLLOWING OF THE ABOVE MENTIONED QUANTITIES
C ARE WRITTEN ON UNIT 7: (1) AND (2), AND WHEN LOUTC IS TRUE (3) - (5)
C AND ELSE (11), (10) AND (5).
      IMPLICIT INTEGER(I,J,K,M,N), LOGICAL(L),REAL *8(A-H,O-Z)
     *COMMON/OUTC/I,I1,I2,I4,I21,I31,IANG,LPUNCH,LOUTC,LNTRAN,LNERNO
     9LK30
      COMMON/OBSER/OBS(20)
      DIMENSION OBN(10)
      IF (DABS(SLAT) .LT. 0.1D-6) SLAT = 0.0D0
      IF (DABS(SLON) .LT. 0.1D-6) SLON = 0.0D0
C THIS IS DONE IN ORDER TO AVOID PRINTING OF SIGN, WHEN THE ARC-SECOND
C PART IS NEAR TO ZERO,(OR ZERO IS REPRESENTED AY A SMALL NEGATIVE NUM-
C BER).
      IF (OBS(1).GE.1.0D4) OBS(1) = 9999.9D0
      IF (OBS(1).LE.-1.0D3)OBS(1) = -999.99D0
      IF (.NOT.LPUNCH) GO TO 8010
      IF (LOUTC) GO TD 8007
      OBN(1) = OBS(1)
      IO = 2
      OBN(2) = OBS(I4)
      IF (LNTRAN) GO TO 8031
      OBN(3) = OBS(I4-1)
      IO = IO+1
 8031 IF (LNERNO) GO TO 8032
      IO = IO+1
      OBN(10) = OBS(I)
 8032 IF (LONC) GO TO 8034
      IO = IO+1
      OBN(IO) = OBS(I31)
      IF (LNTRAN) GC 70 8033
      IO = IO+1
      OBN(IO) = OBS(I4+9)
 8033 IF (LNERNO) GO TO 8034
      IO = IO+1
      OBN(IO) = OBS(I21)
```

```
 8034 I2 = I0
      GO TO 8010
C
 8007 DO 8008 M = 1, I
 8008 OBN(M) = OBS(M)
      IF (LONC) GO TO 8010
      DO 8009 M = 2, I
 8009 OBN(M+I-1) = OBS(M+10)
C
 8010 GO TO (8000,8001,8002,8002),IANG
 8000 WRITE(6,800)NO,IDLAT,MLAT,SLAT,IDLON,MLON,SLON,(OBS(J),J=1,I)
  800 FORMAT(' ',I7,2(I5,I3,F6.2),F8.2,2F7.2,F6.2,4F7.2)
      IF (LPUNCH) WRITE(7,810)NO,IDLAT,MLAT,SLAT,IDLON,MLON,SLON,(OBN(J)
     *, J = 1, I2)
  810 FORMAT(I7,2(I4,I2,F5.2),7F7.2)
      GO TO 8004
 8001 WRITE(6,801)NO,IDLAT,SLAT,IDLON,SLON,(OBS(J),J=1,I)
  801 FORMAT(' ',I7,2(I8,F6.2),F8.2,2F7.2,F6.2,4F7.2)
      IF (LPUNCH) WRITE(7,811)NO,IDLAT,SLAT,IDLON,SLON,(OBN(J),J=1,I2)
  811 FORMAT(I7,2(I5,F6.2),7F7.2)
      GO TO 8004
 8002 WRITE(6,802)NO,SLAT,SLON,(OBS(J),J=1,I)
  802 FORMAT(' ',I7,2F14.9,F8.2,2F7.2,F6.2,4F7.2)
      IF (LPUNCH) WRITE(7,812)NO,SLAT,SLON,(OBN(J), J = 1, I2)
  812 FORMAT(I7,2(F11.6),7F7.2)
C
 8004 IF (LK30) WRITE(6,803)(OBS(J+4), J = 1, I1)
C OUTPUT OF TRA,POT,COLL1,COLL2,PRED,OR PRED+TRA
      IF (LONC) RETURN
C OUTPUT OF OBS,DIFR OR ERR FOR ETA
      IF(I.GT.1) WRITE(6,804)(OBS(J+10),J=2,I)
      IF (LK30.AND.I.GT.1) WRITE(6,803)(OBS(J+14), J = 1, I1)
      IF (I.LE.1.AND.LK30) WRITE(6,805)(OBS(J+14), J = 1, I1)
  804 FORMAT('  ',42X,2F7.2,F6.2)
  803 FORMAT('+ ',62X,6F7.2)
  805 FORMAT('  ',62X,6F7.2)
      RETURN
      END


      SUBROUTINE COMPA(VG,VF)
C THE SUBROUTINE IS USED TO COMPARE OBSERVED AND PREDICTED QUANTITIES.
      IMPLICIT INTEGER(I,J,K,M,N),LOGICAL(L),REAL*8(A-H,O-Z)
      COMMON /OBSER/OBS(20)
      DIMENSION NUM(70),VARI(18)
      DATA NUM,VARI,D0/70*0,19*0.00D0/
C INITIALIZING VARIABLES FOR PREDICTION STATISTICS.
      RETURN
C
```

```
      ENTRY COMPB(IKP,LNGR,LREPEC,LONECO)
      IF (LNGR) GO TO 2041
      SCALE = VG
      GO TO 2042
 2041 SCALE = VF
 2042 SCALE2 = SCALE/2
C
      IF (LONECO) I = IKP-2
      IF (LREPEC) I = 1
      INN = I*22+1
      INV = I*6+1
      RETURN
C
      ENTRY COMPC(IU)
      J=0
 3028 OBS(J+3) = OBS(J+2)-OBS(J+IU)
      DO 3035  I=1,3
      GO TO (3040,3041,3042),I
 3040 OB1 = OBS(J+2)
      GO TO 3043
 3041 OB1 = OBS(J+IU)
      GO TO 3043
 3042 OB1 = OBS(J+3)
C OB1 IS NOW EQUAL TO THE DIFFERENCE BETWEEN MEASURED AND PREDICTED
C QUANTITIES.
C COMPUTATION OF SUM AND SQUARESUM FOR PREDICTION STATISTICS.
 3043 VARI(INV+I-1) = VARI(INV+I-1)+OB1
 3035 VARI(INV+I+2) = VARI(INV+I+2)+OB1**2
C COUNTING NUMBER OF OBS OF TYPE IKP.
      NUM(INN) = NUM(INN)+1
C
      IND = (DABS(OB1)+SCALE2)/SCALE
      IF (OB1 .LT. D0) IND = -IND
      IND = IND+11
      IF(IND.GT.21.OR.IND.LT.1)IND=22
      IND=IND+INN
      NUM(IND) = NUM(IND)+1
      IF (LONECO) GO TO 3029
      IF (INN .EQ. 47) GO TO 3036
      INN = 47
      INV = 13
      J = 10
      GO TO 3028
 3036 INN = 24
      INV = 7
 3029 RETURN
C
      ENTRY OUTCOM
C OUTPUT OF PREDICTION STATISTICS.
      J = 0
```

```
  401 FORMAT('1COMPARISON OF PREDICTIONS AND OBSERVATIONS')
      WRITE(6,401)
      DO 4020 IKP = 1,3
      INN = J*23+1
      NC = NUM(INN)
      IF(NC.EQ.0) GO TO 4020
      INV = J*6+1
      INW = INV+2
      DO 4002 I = INV,INW
 4002 VARI(I) = VARI(I)/NC
      GO TO (4003,4004,4005),IKP
 4003 WRITE(6,402)
  402 FORMAT('0GRAVITY ANOMALIES')
      GO TO 4006
 4004 WRITE(6,403)
  403 FORMAT('0LATITUDE COMPONENT OF DEFLECTION OF THE VERTICAL (KSI)')
      GO TO 4006
 4005 WRITE(6,404)
  404 FORMAT('0LONGITUDE COMPONENT OF DEFLECTION OF THE VERTICAL (ETA)')
C
 4006 WRITE(6,405)NC,(VARI(I),I=INV,INW)
  405 FORMAT(' NUMBER:',I4/
     *'0          OBSERVATIONS PREDICTIONS DIFFERENCE'/
     *' MEAN      ',3F12.2)
      IF(NC.EQ.1) GO TO 4007
      INV = INV+3
      INW = INW+3
      DO 4015 I = INV,INW
 4015 VARI(I) =(VARI(I)-VARI(I-3)**2*NC)/(NC-1)
  406 FORMAT(' VARIANCE ',3F12.2)
      WRITE(6,406)(VARI(I),I = INV,INW)
  407 FORMAT('0DISTRIBUTION OF DIFFERENCES, UNITS:',F6.2)
C
 4007 GO TO (4008,4009,4009),IKP
 4008 WRITE(6,407)VG
      WRITE(6,408)
  408 FORMAT('+ ',45X,'MGAL')
      GO TO 4010
 4009 WRITE(6,407)VF
      WRITE(6,409)
  409 FORMAT('+ ',45X,'ARCSEC')
 4010 INN = INN+1
      NC = INN+20
      WRITE(6,410)(NUM(I),I=INN,NC),NUM(NC+1)
  410 FORMAT(' ',21I3,3X,I5/
     *' -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
     *OUTSIDE')
 4020 J = J+1
      RETURN
      END
```

```
      SUBROUTINE PRED(SS,SREF,UO,AA,IS,ISO,II,IC,NC,IMAX1,LPRED,LBST,
     *LCST)
C THE SUBROUTINE COMPUTES THE COVARIANCES BETWEEN A QUANTITY OF TYPE IKP
C (HAVING COORDINATES RLONGP,RLATP) AND IC OTHER QUANTITIES HAVING COOR-
C DINATES STORED IN THE ARRAYS RLAT,RLONG. INFORMATION ABOUT THE KIND OF
C QUANTITIES ARE FOUND IN THE ARRAY INDEX AND P.
C BECAUSE THE SUBROUTINE MAY BE CALLED SEVERAL TIMES FOR THE SAME TYPE
C OF QUANTITY SOME COMMON VARIABLES ARE TRANSFERRED THROUGH /PR/. THE
C INTEGERS II AND IS GIVES INFORMATION ABOUT FROM WHICH PLACE IN THE
C DIFFERENT ARRAYS THE COORDINATES AND DEGREE-VARIANCES ARE TO BE PICKED
C UP (ACCORDING TO COLL.I OR II). THE COMPUTED COVARIANCES ARE STORED IN
C THE ARRAY C. THUS WHEN LBST IS TRUE, THEY ARE FIRST STORED IN ARRAY B
C AND LATER TRANSFERRED TO C.
C WHEN LCST IS TRUE, THE PROCEDURE IS USED TO COMPUTE EITHER THE COEF-
C FICIENTS OF THE NORMAL EQ. OR THE VECTOR OF COVARIANCES USED IN THE
C COMPUTATION OF THE ERROR OF PREDICTION.
C WHEN LPRED IS TRUE (COMP. OF PREDICTIONS), THE PRODUCT OF THE COVARI-
C ANCES AND THE SOLUTIONS TO THE NORMAL-EQ.(FOUND IN B) ARE ACCUMULATED
C IN THE VARIABLE PREDP (RESP. PRETAP).
      IMPLICIT INTEGER(I,J,K,M,N), LOGICAL(L),REAL *8(A-H,O-Z)
      COMMON /NESOL/C(4700),NCAT(100),ISZE(100),NBL(310),MAXBL,IQ
      COMMON/PR/SIGMA(250),SIGMAO(250),B(1600),P(42),
     *SINLAT(1600),COSLAT(1600),RLAT(1600),RLONG(1600),COSLAP,SINLAP,
     *RLATP,RLONGP,RP,PRETAP,PREDP,PW,LONECO,LNKSIP,LNETAP,LDEFVP,LNDFP,
     *LGRP,LNGR,LKEC1,LKEC3,LKNE1,IV1,NI,NR,KTYPE,INDEX(42)
      COMMON /DCON/DO,D1,D2,D3
      COMMON /SCK/IK,IKO,IK1,IK2,IKA
C
      LIMAX3 = IMAX1.LT.3
      LNBC = .NOT.LBST.AND.LCST
      JR = II
      NR = ISO+1
      JRNEXT = NR
      PRETAP = DO
      PREDP = DO
C
C COMPUTATION OF THE COVARIANCE CORRESPONDING TO OBSERVATION IN P
C AND OBSERVATIONS NUMBER 1 TO IC.
      DO 3019 IR = 1, IC
      IIR = IR+ISO
      COSLAQ = COSLAT(IIR)
      SINLAQ = SINLAT(IIR)
      DLAT = RLATP-RLAT(IIR)
      DLONG = RLONGP-RLONG(IIR)
      SINDLA = DSIN(DLAT/D2)**2
      SIDLO2 = DSIN(DLONG/D2)**2
C
```

```
C T IS COSINE TO THE SPHERICAL DISTANCE BETWEEN P AND Q,CF.REF(B) EQ.
C (57).
      T1 = D2*(SINDLA+COSLAP*COSLAQ*SIDLO2)
      T = D1-T1
C
      IF (IIR.NE.JRNEXT) GO TO 3005
C GETTING INFORMATION ABOUT NEXT DATA-SET FROM CATALOGUE
      JRNEXT = INDEX(JR)
      IKQ = INDEX(JR+1)
      PQW = P(JR)*PW
      SD = RP*P(JR+1)
      S = SS/SD
      A = AA/(SD*SD)
      S2 = S*S
      JR = JR+2
C
      LREPER = IKQ .EQ. 5
      LNDFQ = IKQ.LT.3
      LNDERI = LNDFP.AND.LNDFQ
      LNONDD = LNDFP.OR.LNDFQ
      LGRQ = IKQ.EQ.2
      IF (LGRP.OR.LGRQ) IV = 1
      IF (LGRP.AND.LGRQ) IV = 2
      LIVO = (LNGR.AND.(.NOT.LGRQ))
      IF (LIVO) IV = 0
      IF (IV.EQ.IV1 .AND. DABS(SREF-S).LT.1.0D-8) GO TO 3005
      IVM1 = IV-1
      IV1 = IV
C
      SREF = S
      IF (LIMAX3) GO TO 3005
C COMPUTATION OF DEGREE-VARIANCES OF NEW TYPE IF NECESSARY
      IF (DABS(SD-D1).GT.1.0D-8) GO TO 3003
      DO 3002 I = 3, IMAX1
 3002 SIGMA(I+IS) = SIGMAO(I+IS)*(I-2)**IV
      GO TO 3005
 3003 SI = D1/(SD**3)
      DO 3004 I = 3, IMAX1
      SI = SI/SD
 3004 SIGMA(I+IS) = SIGMAO(I+IS)*(I-2)**IV*SI
 3005 LREROW =LREPER
C
      TS = T*S
      RLL = D1-D2*TS+S2
      RL = DSQRT(RLL)
      RN = D1-TS+RL
      RLNL = DLOG(D2/RN)
C
C SUMMATION OF LEGENDRE SERIES,CF. REF(A) PAGE 66 F.
      M = IMAX1
```

```
      U1 = U0
      A0 = D0
      A1 = D0
      DA0 = D0
      DA1 = D0
      DD0 = D0
      DD1 = D0
      IF (LIMAX3) GO TO 3007
      DO 3006 I = 1,IMAX1
      U2 = U1-D1
      U1 = D2-D1/DFLOAT(M)
      UTS = U1*T
      A2 = A1
      A1 = A0
      A0 = UTS*A1-U2*A2+SIGMA(M+IS)
      M = M-1
      IF(LNDERI)GO TO 3006
C FIRST DERIVATIVES OF SERIES
      A2 = DA1
      DA1 = DA0
      DA0 = UTS*DA1-A2*U2+A1*U1
      IF(LNONDD)GO TO 3006
C SECOND DERIVATIVE OF SERIES
      A2 = DD1
      DD1 = DD0
      DD0 = UTS*DD1-A2*U2+D2*DA1*U1
 3006 CONTINUE
C
 3007 DML = D1-RL
      IF (LKEQ1) GO TO 3120
      T2 = T*T
      P2 = (D3*T2-D1)/D2
      SP2 = S2*P2
      DTS = D3*TS+D1
      RM = DML-TS
      RM2 = RM/2
      IF (LKEQ3) CALL SUMK(S,S2,RL,RN,T,RL2,DRL2,DDRL2,LNDERI,LNONDD)
C JUMP TO COVARIANCE NOT INVOLVING DEFLECTIONS
 3120 IF (LNDERI) GO TO 3014
      DPL = D1+RL
      RNL = RL*RN
      IF (LKEQ3) DKC = (DTS/RL+D2-7.0D0*TS-D3*RL)/D2
     *                     +D3*TS*RLNL+SP2*DPL/RNL
C BELOW COMPUTATION OF FIRST AND SECOND DERIVATIVES OF CLOSED PARTS OF
C K(P,Q) AND COV(DELTA G(P),T(Q)) WITH RESPECT TO THE VARIABLE T, CF.
C REF(A) SECTION 8, EQ. (108)-(110), (119)-(121) AND (133)-(135).
C
      IF (IVM1) 3009,3008,3010
 3008 GO TO (3121,3122,3123),KTYPE
 3121 RLO = DPL/RNL-D1
```

```
      GO TO 3130
 3122 RLO = (DML*DTS/RL-TS)/D2+D3*RM2+D3*TS*RLNL+SP2*DPL/RNL
      GO TO 3130
 3123 RLO = (DKC-DRL2)/IK2
      GO TO 3130
C
 3009 GO TO (3126,3127,3128),KTYPE
 3126 RLO = TS/RN+RLNL
      GO TO 3130
 3127 RLO = (D3*TS-D1)*RLNL+(SP2-TS)*DPL/RNL+TS+DML*(D1-D3*RM2)/RL
      GO TO 3130
 3128 DKB = DML/RL-D3*TS+RLNL+TS*DPL/RNL
      RLO = (IK1*DKC-IK2*DKB+DRL2)/IKA
 3130 RLO = DAO+A*S*RLO
      IF (LNONDD) GO TO 3010
C
      IF (LKNE1) RL3 = RLL*RL
      GO TO (3131,3132,3133),KTYPE
 3131 RL1 = (D1+DPL**2/RNL)/RN
      GO TO 3135
 3132 RL1 = (D3*TS-D1)*(D2*DPL/RNL+D1/(D2*RL3))+D3*(RLNL+D1/RL)
     *        +(SP2-TS)*(DPL*DPL*RL+RN)/(RN**2*RL3)-D1/D2
      GO TO 3135
 3133 DDKC = (DTS/RL3-7.0D0)/D2+D3*(RLNL+D1/RL)+6.0D0*TS*DPL/RNL
     *          +SP2*((DPL/PNL)**2+D1/(RL3*RN))
      DDKB = D1/RL3-D3+D2*DPL/RNL+TS*((DPL/RNL)**2+D1/(RL3*RN))
      RL1 = (IK1*DDKC-IK2*DDKB+DDRL2)/IKA
 3135 RL1 = DD0+A*S2*RL1
C
 3010 LKSIQ = LREPER .OR. IKQ .EQ. 3
      LETAQ = IKQ .EQ. 4
      SINDLO = DSIN(DLONG)
      SINDLA = DSIN(DLAT)
C  THE DERIVATIVE OF  T  WITH RESPECT TO THE LATITUDE IN P AND Q.
      IF (.NOT.LNKSIP) DLAP = -SINDLA+D2*SINLAP*COSLAQ*SIDLO2
      IF (LKSIQ) DLAQ = SINDLA+D2*COSLAP*SINLAQ*SIDLO2
C
 3011 IF(LNETAP)GO TO 3012
C  COVARIANCE BETWEEN ETAP AND OTHER FUNCTIONALS IN Q.
      IF(LNONDD)COVPQ = RLO*SINDLO*COSLAQ
      IF(LKSIQ) COVPQ = SINDLO*(SINLAQ*RLO-DLAQ*COSLAQ*RL1)
      IF(LETAQ) COVPQ = -SINDLO**2*COSLAQ*COSLAP*RL1+DCOS(DLONG)*RLO
      IF (LONECO) GO TO 3012
      IF(LBST) B(NR)=COVPQ/PQW
      IF (LNBC) C(NI+NC) = COVPQ/PQW
      IF(LPRED) PRETAP = PRETAP+B(NR)*COVPQ
 3012 IF(LNKSIP)GO TO 3013
C  COVARIANCE BETWEEN KSIP AND OTHER FUNCTIONALS IN Q.
      IF(LNONDD)COVPQ = -RLO*DLAP
      IF (LKSIQ) COVPQ = DLAP*DLAQ*RL1+(DCOS(DLAT)
```

```
     *-D2*SINLAP*SINLAQ*SIDLO2)*RLO
      IF(LETAQ)COVPQ = (DLAP*COSLAP*RL1-SINLAP*RLO)*SINDLO
C
 3013 IF(LDEFVP)GO TO 3018
C  COVARIANCE BETWEEN KSIQ,ETAQ AND GRAVITY OR HEIGHT ANOMALY IN P.
      IF(LKSIQ)COVPQ = -RLO*DLAQ
      IF(LETAQ)COVPQ = -RLO*SINDLO*COSLAP
      GO TO 3018
C
 3014 IF (LKEQ1) GO TO 3034
      RKC = DTS*RM2+((D1-T2)/4.0D0+P2*RLNL)*S2
      RKB = RM-SP2+TS*RLNL
 3034 IF (IVM1) 3015,3016,3017
C CF.REF(A), EQ.(105),(115) AND (130).
 3015 GO TO (3141,3142,3143),KTYPE
 3141 RLO = DML+(TS-D1)*RLNL
      GO TO 3155
 3142 RLO = RKC-RKB
      GO TO 3155
 3143 RLO = (IK1*RKC-IK2*RKB+RL2)/IKA
      GO TO 3155
C
C CF.REF(A), EQ.(106),(116) AND (131).
 3016 GO TO (3146,3147,3148),KTYPE
 3146 RLO = RLNL -TS
      GO TO 3155
 3147 RLO = RKC
      GO TO 3155
 3148 RLO = (RKC-RL2)/IK2
      GO TO 3155
C
C CF.REF(A), EQ.(107),(117) AND (132).
 3017 GO TO (3151,3152,3153),KTYPE
 3151 RLO = DML/RL-RLNL
      GO TO 3155
 3152 RLO = DML/RL-TS-SP2+RKC
      GO TO 3155
 3153 RLO = (RKC+IK1*RL2)/IK2
C
 3155 COVPQ = A0+A*RLO
C
 3018 IF(LCST)C(NI) = COVPQ/PQW
      IF(LPRED)PREDP = PREDP+COVPQ*B(NR)
      NI = NI+1
      NR = NR+1
      IF (.NOT.LREROW.OR.(IC.EQ.IR.AND.(.NOT.LPRED))) GO TO 3019
      LKSIQ = .FALSE.
      LETAQ = .TRUE.
      LREROW = .FALSE.
      GO TO 3011
```

```
 3019 CONTINUE
C END OF LOOP COMPUTING A NR-1 VECTOR OF COVARIANCES.
      RETURN
      END



      SUBROUTINE SUMK(S,S2,RL,TSL,T,RL2,DRL2,DDRL2,LND,LNDD)
C COMPUTATION OF THE SUM OF AN INFINITE LEGENDRE SERIES AND ITS FIRST
C AND SECOND DERIVATIVES WITH RESPECT TO THE VARIABLE T, CF. REF(A) EQ.
C (72),(96)-(104). THE SUM DIVEDED BY S IS RETURNED BY THE VARIABLE
C RL2, THE FIRST DERIVATIVE (DIVEDED BY S*S=S2) BY DRL2 AND THE SECOND
C DERIVATIVE (DIVEDED BY S*S2) BY DDRL2.
C THE SUM IS THE SUM OF THE TERMS 1/(I+IK)*S**(I+1)*(LEGENDRE POLYNOMIAL
C OF ORDER I,EVALUATED IN T) FROM THREE TO INFINITY.
      IMPLICIT INTEGER(I,J,K,M,N), LOGICAL(L),REAL *8(A-H,O-Z)
      COMMON /SCK/IK,IKO,IK1,IK2,IKA
      COMMON /DCON/DO,D1,D2,D3
      RLO = DLOG(D1+D2*S/(D1-S+RL))
      RL1 = (RL-D1+T*RLO)/S2
      RLI = RL/S2
      RLO = RLO/S
      IF (LND) GO TO 901
      DRLI = -D1/(RL*S)
      DRLO = S/(TSL*RL)
      DRL1 = (-D1/RL+T*DRLO+RLO)/S
      IF (LNDD) GO TO 901
      RL3 = RL**3
      DDRLI = -D1/RL3
      DDRLO = S2*((D1+RL)/((TSL*RL)**2)+D1/(TSL*RL3))
      DDRL1 = -D1/RL3+(D2*DRLO+T*DDRLO)/S
C
  901 DO 902 I = 2, IKO
      RI = DFLOAT(I)
      D21 = (D2*RI-D1)/S
      D11 = (RI-D1)/S2
      RL2 = (RLI+D21*T*RL1-D11*RLO)/RI
      RLO = RL1
      RL1 = RL2
      IF (LND) GO TO 902
C  FIRST ORDER DERIVATIVE WITH RESPECT TO T.
      DRL2 = (DRLI+D21*(RLO+T*DRL1)-D11*DRLO)/RI
      DRLO = DRL1
      DRL1 = DRL2
      IF (LNDD) GO TO 902
C  SECOND DERIVATIVE WITH RESPECT TO T.
      DDRL2 = (DDRLI+D21*(D2*DRLO+T*DDRL1)-D11*DDRLO)/RI
      DDRLO = DDRL1
      DDRL1 = DDRL2
  902 CONTINUE
```

```
      RL2 = RL2-D1/IK-S*T/IK1-(D3*T*T-D1)*S2/(D2*IK2)
      IF (LND) GO TO 905
      DRL2 = DRL2/S-D1/IK1-D3*S*T/IK2
      IF (LNDD) GO TO 905
      DDRL2 = DDRL2/S2-D3/IK2
905   RETURN
      END
```

Appendix B.

An input example.

The digital numbers written to the left of the input data correspond to the numbers identifying the input specification given in Section 6.

The input example will produce the output shown in Appendix C. It corresponds to the situation described in Section 4, i.e. where three datasets of observations x_1, x_2, and x_3 are used for the determination of $\tilde{T}$.

```
1.0     T T T T T
1.1     (' EUROPEAN DATUM, 1950.')
1.2      6378388.0 297.0      T
1.3      6378143.0 3.98601220D 14 298.25      5.20D-06
         -111.1 -111.5 -142.6  1.57  0.19  1.05 T
1.31     55 30 00.00 10 00 00.00  -1.00   1.00   0.00
2.0     (' R.H.RAPP, THE GEOPOTENTIAL TO (14,14) ETC., IUGG, LUCERNE, 1967.')
2.1      3.98601220D 14  6378143.0 -484.1803   8 T
2.2     (9F8.4)
2.12      0.0000  0.0000  2.3241 -1.3106  0.9025  1.7842  0.2601  0.7224 -0.5755
          0.7240  1.3860  0.5608 -0.5289 -0.4290  0.1966  0.5120  0.8804 -0.2317
          0.1861  0.3065  0.0296 -0.0670 -0.0161  0.3934 -0.1339 -0.1203 -0.0209
          0.1371  0.2119  0.0317 -0.5727 -0.0703 -0.0834 -0.0062  0.0153 -0.2185
         -0.0486  0.0774 -0.0300 -0.4660 -0.2760 -0.4555 -0.0124 -0.2009  0.0890
          0.0996  0.0062  0.2632  0.0832  0.1540 -0.0391 -0.2898 -0.0145  0.0612
          0.0634 -0.1685  0.1512  0.0897 -0.0672 -0.0195 -0.0738  0.0017  0.0621
          0.2220  0.0262  0.0907 -0.0029  0.0568 -0.0757  0.0702 -0.0859  0.2170
          0.0246  0.0668 -0.0948 -0.1090
3.0      3
3.01      24
3.1      0.9998  1600.00   0
3.11    12
3.12    (11F5.1)
3.13      0.0  0.0  0.0  0.0  0.0  0.0  0.0 15.1 17.7 13.7  8.4
4.0     (16,13,F5.1,13,F5.1,F5.1,L1)
4.1       1  2  3  2  0  4  0  2 1.0015561 F F T T F
4.11         1.0         0.00
4.12      4.00
4.2     937269 56 30.0  9 00.0 18.0
        937271 56 30.0 11 00.0 10.0
        937273 56 30.0 13 00.0 27.0
        937289 55 30.0  9 00.0 26.0
        937291 55 30.0 11 00.0 12.0
        937293 55 30.0 13 00.0  6.0
        937309 54 30.0  9 00.0  5.0
        937311 54 30.0 11 00.0 10.0
        937313 54 30.0 13 00.0  2.0T
4.3     T T F
```

```
4.31      9   10  0.2196870D+00  0.4327501D+02
        -0.9830116129694D-01-0.5712695713083D+00 0.7692296486919D+00 0.8015281184466D+00
        -0.4445062511601D-01-0.5063771197996D+00-0.7147287183765D+00 0.5106668018712D-01
        -0.5062347113640D+00 0.1785338257807D+01
4.4     T
3.1      0.999800 170.00  90
4.0     (I5,20X,I3,I3,F6.2,I3,I3,F6.2,2F7.2,L1)
4.1       1  2  3  1  0  4  5  5 1.0        F F F T F
4.12      0.30
4.2     40216 EJER BAVNEHOEJ      55 58 39.68  9 49 54. 0  -5.13   3.61
        40621 ST MAGLEHOEJ        55 57 56.16 12 02 20. 7  -2.20  -0.37
        40058 HOEGEBJERG          54 58 01.40  9 58 32. 8  -5.13   1.11
        40606 VIGERLOESE          54 42 54.22 11 55 55.39  -1.34   1.00T
4.3     F
4.0     (I3,2(2I3,F6.2),F6.2,F5.2,L1)
4.1       1  2  3  1  0  4  0  1 1.0        F F F F F
4.2      28 55 52 38.51 12 50 38.93 20.40 0.00T
4.3     F
4.0     (I6,2(I3,F6.2),2F7.2,L2)
4.1       1  2  3  2  4  5  0  2 1.0        F F F T F
4.11          1.00 981000.00 T
4.12      0.20
4.2     261301 56 04.90 10 00.34  70.60 621.54
        700181 56 07.10 11 57.00   0.00 603.20
          4413 54 51.49  9 59.55   6.94 517.58
          3163 54 50.23 12 00.01  17.63 510.18T
4.3     T F F
```

```
5.0       F T T
5.01           1.00      0.10
5.030     (I5,20X,I3,I3,F6.2,I3,I3,F6.2,2F7.2,L1)
5.031       1  2  3  1  0  4  5  5 1.0        F F F F F
5.033     40216 EJER BAVNEHOEJ        55 58 39.68  9 49 54.90  -5.13   3-61
          40606 VIGERLOESE            54 42 54-22 11 55 55.39  -1.34   1.00T
5.1       F
5.0       F T T
5.030     (I6,2(I3,F6.2),2F7.2,L2)
5.031       1  2  3  2  4  5  0  2 1.0        F F F F F
5.032          1.00 981000-00 T
5.033       4413 54 51.49  9 59.55   6-94 517.58
            3163 54 50.23 12 00.01  17.63 510.18T
5.1       F
5.0       T T F
             54 30.00    9 00.00   30.00  60.00   1   1 2 1.0156961 F F F
5.1       F
5.0       T T F
5.02         54 30.00    9 00.00   30.00  60.00   1   1 1 1.0156961 F F F
5.1       F
5.0       T T F
5.02         54 30.00    9 00.00   30.00  60.00   1   1 5 1.0156961 F F F
5.1       F
5.0       T T F
5.02         54 30.00    9 00.00   30.00  60.00   2   2 1 1.0       T F F
5.1       T
```

Appendix C.

An output example.

The output has been produced by the FORTRAN program using the input given in Appendix B.

GEODETIC COLLOCATION,VERSION 20 APR 1974.

MOTE THAT THE FUNCTIONALS ARE IN SPHERICAL APPROXIMATION
MEAN RADIUS = RE = 6371 KM AND MEAN GRAVITY 981 KGAL USED.

LEGEND OF TABLES OF OBSERVATIONS AND PREDICTIONS:
OBS = OBSERVED VALUE (WHEN BOTH COMPONENTS OF DEFLECTIONS ARE OBSERVED ETA BELOW KSI), DIF =THE DIFFERENCE BETWEEN OBSERVED AND PREDICTED VALUE, WHEN PREDICTIONS ARE COMPUTED AND ELSE THE RESIDUAL OBSERVATION, ERR = ESTIMATED ERROR OF PREDICTION TRA = CONTRIBUTION FROM DATUM TRANSFORMATION, POT = CONTRI-BUTION FROM POTENTIAL COEFFICIENTS, COLL = CONTRIBUTION FROM COLLOCATION DETERMINED PART OF ESTIMATE, WHEN THERE ONLY HAS BEEN USED ONE SET OF OBSERVATIONS (DIFFERENT FROM POT.COEFF.) COLL1 = CONTRIBUTION FROM ESTIMATE OF ANOMALOUS POT. DETER-MINED FROM FIRST SET OF OBSERVATIONS, COLL2 = CONTRIBUTION FROM ESTIMATE OBTAINED FROM SECOND SET OF OBSERVATIONS, PRED= PREDICTED VALUE IN NEW REFERENCE SYSTEM, WHEN PREDICTIONS ARE COMPUTED AND ELSE THE SUM OF THE CONTRIBUTIONS FROM THE POT. COEFFICIENTS AND FIRST ESTIMATE OF AYOMALDUS POTENTIAL, AND PRED-TRA = PREDICTION OR SUM OF CONTRIBUTIONS IN THE OLD RE-FERENCE SYSTEM.

REFFREYCE SYSTEM:
EUROPEAN DATUM. 1950.

A = 6378388.0 M
1/F = 297.00040
REF.GRAVITY AT EQUATOR = 978049.00 MGAL
POTENTIAL AT REF.ELL. = 62639787.00 M**2/SEC**2
GRAVITY FORMULA:
INTERNATIONAL GRAVITY FORMULA* POTSDAM SYSTEM.

NEW A	NEW GM	NEW 1/F
6378143.0	0.390601 2D+15	298.25000

DL	DX	DY	DZ
0.52D-05	-111.1	-111.5	-142.6

EPS1	EPS2	EPS3
1.57	0.19	1.05

NEW REF. GRAVITY AT EQUATOR= 97R032.67 MGAL
NEW POTENTIAL AT ELLIPSOID = 62636916.84 M**2/SEC**2

DEFLECTIONS AND HEIGHT ANOMALIES CHANGED IN

LATITUDE			LONGITUDE			BY DKSI	DETA	DZETA
55	30	0.0	10	0	0.0	-1.00	1.00	0.0

SOURCE OF THE POTENTIAL COEFFICIENTS USED:
R.H.RAPP, THE GEOPOTENTIAL TO (14,14) ETC., IUGG, LUCERNE, 1967.

GM	A	COFF(5)	MAX.DEGREE
0.39860122D+15	6378143.0	-484.1803	8

START OF COLLOCATION I:

THE MODEL ANOMALY DEGREE-VARIANCES ARE EQUAL TO
A - ((I-2)*(I+ 24)).

RATIO R/RE = 0.999800
VARIANCE OF POINT GRAVITY ANOMALIES = 1600.00 MGAL**2
THE FACTOR A = 412.88 MGAL**2
12 EMPIRICAL ANOMALY DEGREE-VARIANCES FOR DEGREE > 1,
(IN UNITS OF MGAL**2):
0.0 0.0 0.0 0.0 0.0 0.0 0.0 15.1 17.7 13.7 8.4

OBSERVATIONS:

THE FOLLOWING QUANTITIES ARE MEAN-VALUES, AND ARE REPRESENTED AS POINT VALUES IN A HEIGHT R.

ST.VAR. = 27.98, RATIO R/RE = 1.0015554,

NO	LATITUDE D	M	LONGITUDE D	M	H M	DELTA G (MGAL) OBS	DIF	TRA	POT	POT-TRA
937269	56	30.00	9	0.0	0.0	18.00	-5.26	-6.69	16.57	23.26
937271	56	30.00	11	0.0	0.0	10.00	-13.19	-6.69	16.50	73.19
937273	56	30.00	13	0.0	0.0	27.00	3.93	-6.69	16.37	23.07
937289	55	30.00	9	0.0	0.0	26.00	3.50	-6.47	16.02	22.50
937291	55	30.00	11	0.0	0.0	12.00	-10.43	-6.48	15.96	22-43
937293	55	30.00	13	0.0	0.0	6.00	-16.32	-6.48	15.84	22.32
937309	54	30.00	9	0.0	0.0	5.00	-16.67	-6.25	15.42	21.67
937311	54	30.00	11	0.0	0.0	10.00	-11.61	-6.25	15.36	21.61
937313	54	30.00	13	0.0	0.0	2.00	-19.51	-6.25	15.25	21.51

STANDARD DEVIATIONS OF THE OBSERVATIONS IN THE SAME SEQUENCE
AND IN THE SAME UNITS AS THE OBSERVATIONS:
4.00 4.00 4.00 4.00 4.00 4.00 4.00 4.00 4.00

SOLUTIONS TO NORMAL EQUATIONS:

-0.9830116130D-01-0.5712695713D+00 0.7692296487D+00 0.8015781184D+00-0.4445062512D-01
-0.5063771198D+00-0.7147287184D+00 0.5106668019D-01-0.5062347114D+00

THE SOLUTIONS HAVE BEEN COMPUTED IN A PREVIOUS RUN.

NUMBER OF EQUATION = 9
NORMALIZED SQUARE-SUM OF OBSERVATIONS = 0.178534D+01
NORMALIZED DIFFERENCE BETWEEN SQUARE-SUM OF
OBSERVATIONS AND NORM OF APPROXIMATION = 0.219687D+00

START OF COLLOCATION II:

RATIO R/RE = 0.999800
VARIANCE OF POINT GRAVITY ANOMALIES = 170.00 MGAL**2
THE FACTOR A = 65.83 MGAL**2
90 DEGREE-VARIANCES EQUAL TO ZERO

OBSERVATIONS:

KSI/ETA (ARCSEC) ST.VAR. = 1.95, RATIO R/RE = 1.0000000,

NO	LATITUDE D M S	LONGITUOE O H S	H M	OBS	OIF	TRA	POT	COLL	PRED	PRED-TRA
40216	55 58 39.68	9 49 54.90	0.0	-5.13	-3.99	0.86	-0.07	-0.20	-0.28	-1.14
				3.61	4.21	3.93	1.71	1.62	3.33	-0.60
40621	55 57 56.16	12 2 20.37	0.0	-2.20	0.66	0.78	-0.13	-1.95	-2.08	-2.86
				-0.37	1.97	3.80	1.98	-0.52	1.47	-2.34
40058	54 58 1.40	9 58 32.48	0.0	-5.13	-2.42	1.00	-0.25	-1.45	-1.71	-2.71
				1.11	2.33	3.96	1.72	1.01	2.73	-1.22
40606	54 42 54.22	11 55 55.39	0.0	-1.34	0.28	0.97	-0.34	-0.32	-0.66	-1.62
				1.00	2.11	3.85	1.96	0.78	2.74	-1.11

ZETA (M) ST.VAR. = 0.37, RATIO R/RE = 1.0000000,

NO	LATITUOE O M S	LONGITUDE D M S	H H	OBS	DIF	TRA	POT	COLL	PQED	PRED-TRA
28	55 52 38.51	12 50 38.93	0.0	20.40	-0.07	22.55	46.20	-3.18	43.02	20.47

DELTA G (MGAL) ST.VAR. = 13.04, RATIO R/RE = 1.0000000,

NO	LATITUDE D M	LONGITUOE O H	H H	OBS	DIF	TRA	POT	COLL	PRED	PRED-TRA
261301	56 4.90	10 0.34	70.60	37.49	21.61	-6.60	16.32	-7.04	9.28	15.88
700181	56 7.10	11 57.00	0.0	-5.70	-19.90	-6.61	16.25	-8.66	7.59	14.20
4413	54 51.49	9 59.55	6.94	17.20	5.69	-6.33	15.61	-10.44	5.18	11.51
3163	54 50.23	12 0.01	17.63	14.89	8.68	-6.33	15.52	-15.64	-0.12	6.21

STANDARD DEVIATIONS OF THE OBSERVATIONS IN THE SAME SECUFNCE
AND IN THE SAME UNITS AS THE OBSERVATIONS:

```
0.30   0.30   0.30   0.30   0.30   0.30   0.30   0.30   0.0   0.20
0.20   0.20   0.20
```

COEFFICIENTS OF NORMAL-EQUATIONS, BLOCK 1

```
 1.0237  0.0      1.0237  0.0512  0.0032  1.0237  0.0005 -0.1358  0.0     1.0237
-0.1328  0.0182  -0.0391 -0.0662  1.0237  0.0181  0.0907 -0.0632 -0.0597  0.0
 1.0237 -0.0479   0.0488 -0.1348 -0.0090  0.0547  0.0440  1.0237  0.0461 -0.0426
-0.0089  0.0482   0.0419 -0.1278  0.0     1.0237  0.0009 -0.0240  0.0746 -0.3934
 0.0099  0.0168  -0.1508 -0.0662  1.0000 -0.2780 -0.2589 -0.0105  0.0898 -0.0968
-0.0015  0.0080  -0.0062 -0.0966  1.0002 -0.0106 -0.0787 -0.3605  0.1171 -0.0051
-0.0048 -0.0378  -0.0003  0.2441 -0.0298  1.0002  0.0945 -0.0078  0.0047  0.0051
 0.3743 -0.0333  -0.0132  0.0929 -0.0877 -0.0413 -0.0462  1.0002 -0.0014  0.0016
 0.0936  0.0019   0.0081 -0.0835 -0.3611 -0.1160 -0.0386 -0.0461 -0.0444 -0.0365
 1.0002 -2.0488   2.1613  0.3411  1.0083 -1.2433  1.1969  0.1459  1.0804 -0.1945
 1.6571 -1.5266   0.4366  0.6654 19.9161
```

SOLUTIONS TO NORMAL EQUATIONS:

```
-0.2001116036D+01 0.2635906503D+01-0.3761142414D+00 0.1916345175D+01-0.1909074891D+01
 0.1347093589D+01 0.5735323703D+00 0.1541918057D+01 0.1638187539D+01 0.1649903595D+01
-0.1907160643D+01 0.1433248597D+01 0.1310638851D+01
```

NUMBER OF FCUATIONS = 13
NORMALIZED SQUARE-SUM OF OBSERVATIONS = 0.199161D+02
NORMALIZED DIFFERENCE BETWEEN SQUARE-SUM OF
OBSERVATIONS AND NORM OF APPROXIMATION =-0.424467D+01

PREDICTIONS:

KSI/ETA (ARCSEC) ST.VAR. = 1.95, RATIO R/RE = 1.0000000,

NO	LATITUDE D M S	LONGITUDE D M S	H M	OBS	DIF	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
40216	55 58 39.68	9 49 54.90	0.0	-5.13	-0.09	0.30	0.86	-0.07	-0.20	-3.90	-4.18	-5.04
				3.61	0.12	0.30	3.93	1.71	1.62	4.09	7.42	3.49
40606	54 42 54.22	11 55 55.39	0.0	-1.34	0.03	0.30	0.97	-0.34	-0.32	0.26	-0.40	-1.37
				1.00	0.07	0.30	3.85	1.96	0.78	2.03	4.78	0.93

DELTA G (MGAL) ST.VAR. = 13.04, RATIO R/RE = 1.0000000,

NO	LATITUDE D M	LONGITUDE D M	H M	OBS	DIF	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
4413	54 51.49	9 59.55	6.94	17.20	0.00	0.20	-6.33	15.61	-10.44	5.69	10.86	17.20
3163	54 50.23	12 0.01	17.63	14.89	0.00	0.20	-6.33	15.52	-15.64	8.67	8.55	14.88

DELTA G (MGAL) ST.VAR. = 0.90, RATIO R/RE = 1.0156956,

NO	LATITUDE D M	LONGITUDE D M	H KM	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
1	54 30.00	9 0.0	100.00	0.80	-4.31	13.60	-6.56	-0.24	6.81	11.12
2	54 30.00	10 0.0	100.00	0.74	-4.31	13.57	-7.27	-0.25	6.05	10.36
3	55 0.0	9 0.0	100.00	0.74	-4.40	13.85	-5.46	0.41	8.80	13.20
4	55 0.0	10 0.0	100.00	0.67	-4.40	13.82	-6.49	0.29	7.62	12.01

ZETA (M) ST.VAR. = 0.06, RATIO R/RE = 1.0156956,

NO	LATITUDE D M	LONGITUDE D M	H KM	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
1	54 30.00	9 0.0	100.00	0.05	27.80	46.69	-2.48	-0.01	44.20	16.40
2	54 30.00	10 0.0	100.00	0.05	26.55	46.17	-2.65	-0.01	43.51	16.96
3	55 0.0	9 0.0	100.00	0.05	27.50	46.73	-2.38	0.03	44.38	16.88
4	55 0.0	10 0.0	100.00	0.04	26.27	46.22	-2.55	0.02	43.68	17.41

KSI/ETA (ARCSEC) ST.VAR. = 0.14, RATIO R/RE = 1.0156956,

NO	LATITUDE D M	LONGITUDE D M	H KM	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
1	54 30.00	9 0.0	100.00	0.12	1.10	-0.19	-0.25	-0.11	-0.56	-1.66
				0.12	4.03	1.57	0.60	-0.02	2.16	-1.87
2	54 30.00	10 0.0	100.00	0.11	1.07	-0.22	-0.21	-0.09	-0.52	-1.58
				0.12	3.97	1.69	0.44	0.01	2.14	-1.83
3	55 0.0	9 0.0	100.00	0.12	1.03	-0.12	-0.49	-0.16	-0.76	-1.80
				0.12	4.01	1.57	0.61	-0.04	2.15	-1.86
4	55 0.0	10 0.0	100.00	0.11	1.00	-0.14	-0.46	-0.13	-0.73	-1.72
				0.11	3.95	1.69	0.51	0.07	2.26	-1.69

ZETA (M) ST.VAR. = 0.37, RATIO R/RE = 1.0000000,

NO	LATITUDE D M	LONGITUDE D M	H M	ERR	TRA	POT	COLL1	COLL2	PRED	PRED-TRA
1	54 30.00	9 0.0	0.0	0.34	27.80	48.21	-3.57	-0.10	44.55	16.75
2	54 30.00	10 0.0	0.0	0.31	26.55	47.69	-3.65	-0.15	43.89	17.34
3	54 30.00	11 0.0	0.0	0.30	25.33	47.13	-3.80	-0.04	43.29	17.96
4	55 0.0	9 0.0	0.0	0.32	27.50	48.28	-3.02	0.17	45.43	17.93
5	55 0.0	10 0.0	0.0	0.24	26.27	47.77	-3.36	0.20	44.61	18.34
6	55 0.0	11 0.0	0.0	0.29	25.07	47.21	-3.67	-0.05	43.49	18.42
7	55 30.00	9 0.0	0.0	0.31	27.23	48.33	-2.43	0.37	46.27	19.05
8	55 30.00	10 0.0	0.0	0.27	26.01	47.82	-3.00	0.19	45.00	18.99
9	55 30.00	11 0.0	0.0	0.30	24.82	47.27	-3.44	-0.11	43.73	18.90

```
MAP OF PREDICTIONS:
COORDINATES OF SOUTH-WEST CORNER
 LATITUDE  LONGITUDE AND SIDE LENGTH LAT.   LONG.
   D  M        D  M        M        M
 54 30.00    9  0.0      30.00    60.00


 1904 1899 1890

 1792 1833 1842

 1675 1733 1795


COMPARIZON OF PREDICOIONS AND OBSERVATIONS

GRAVITY ANOMALIES
NUMBER:    Z

         OBSE VATIONS PREDICTIONS DICFERENCE
MEAN             16.04       16.04        0.00
VARIANCE          2.68        2.68        0.00

DISTRIBUTION OF DIFFERENCES  UNITS   1.00      MGL
  0   0   0   0   0   0   0   0   0   0  2  0  0  0  0   0  0  0  0  0  0       0
-10  -9  -8  -7  -6  -5  -4  -3  -2   1  0  1  2  3  4   5  6  7  8  9 10 OUTSIDE

LATITUDE COMPONENT OF DEFLECTION OF THE VERTICA L(KSI I
NUMBER:    2

         OBSERVATIONS PREDICTIONS DIFFERENCE
MEAN            -3.23       -3.20       -0.0:3
VARIANCE         7.18        6.74        0.011

DISTRIBUOIO  OF DIFFERENCES  UNITS  0.10     ARCSEC
  0   0  0 0   0  0  0  0    1  1  0  0  0   0 0  0  0  0  0  0       0
-10  -9 -8 7  -6 -5 -4 -3   -1  0  1  2  3   4 5  6  7  8  9 10 OUTSIDE

LONGITUDE COMPONENT OF DEFLECTION OF THE VERTCAL (ET )
NUMBER:    2

         OSE  VATIONS PREDICTIONS D FFERENE E
MEAN             2.30        2.21        0 10
VARIANCE         3.41        3.28        0 00

DISTRIBUTION OF DIFFERENCES  UNIT     0 0      ARCSE
  0   0      0  0   0  0  0  0    0  0  2 0    0 0  0  0  0  0  0  0       0
-10  -9  8  -7 -6  -5 -4 -3 -2 -  1  0  1 2    3 4  5  6  7  8  9 10 OUTSIDE
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